Calculus Lab 34—Polar Coordinates

Objective: To understand and be able to interpret graphs of polar coordinate equations.

Maple Commands:

`with(plots):` Loads the `plots` package.

`polarplot(expr,theta=a..b);` Plots a polar coordinate expression $r=f(\theta)$=`expr`.

`polarplot([expr1,expr2,t=a..b]);` Plots parametric equations $r=f(t)$=`expr1` and $\theta=g(t)$=`expr2` in polar coordinates.

Let's try a couple of examples of the above commands:

```
polarplot(1+4*cos(6*theta),theta=0..2*Pi,scaling=
constrained);
polarplot([sin(t),cos(t),t=0..2*Pi]);
```

To answer the following questions, it may help to remember that the slope of the tangent line to a curve whose equation is written $r=f(\theta)$ is given by the formula

$$slope = \frac{r'(\theta)\sin\theta + r(\theta)\cos\theta}{r'(\theta)\cos\theta - r(\theta)\sin\theta}$$

Exercise 1: Consider the family of curves that obey the equation

$$r(\theta) = 1 + a\sin(\theta)$$

where a is a parameter. Plot several examples of this family for different values of a between -2 and 2. For what values of a do these curves pass through the origin? Can you explain this behaviour? For what values of a do these curves have a cusp? Again, can you give an explanation for this behaviour?

Exercise 2: Consider the curves given by $r = \sin(m\theta)r$, where m is an integer. Plot examples of these curves for 4 different values of m between 1 and 10. Sketch an example to hand in. How does the number of loops depend on the value of m?

Find a formula for the slope of the tangent line to the curve $r = r(\theta)$ each time the curve passes through the origin (hint: set r=0 in the above formula for slope). Can you prove the relationship you already found between the number of loops in the curve and the value of m? [Hint: Ask yourself how many different values of θ lead to r=0. Then ask yourself if the slope of the curve (as it passes through

r=0) is different for each of these different values of θ. For each different slope, there will be a new loop.]