Calculus Lab 32—3D Plotting

Objective: To graph some simple surfaces using the common coordinate systems of 3-dimensional space, especially spherical polar and cylindrical coordinates.

Notation: Your textbook uses r for the cylindrical coordinate given by $r=\sqrt{x^2+y^2}$ and uses ρ for the spherical radial coordinate $\rho=\sqrt{x^2+y^2+z^2}$. We will do the same here, writing out `rho` for ρ where necessary.

To begin, go to the `Options` menu, drag down to `Plot Display`, and choose `Window` from the pop-up menu at the side. Plots will now display in their own separate windows, which we will be able to control. Next, load the full `plots` package:

```
with(plots):
```

Let's now try a few 3d plots. They will appear in their own windows. Notice you can drag the image within the window to view the graph from different angles. Try it.

```
plot3d(2,x=-2..2,y=-2..2,scaling=constrained,
axes=normal);
```
Plots the plane z=2. Try dragging the image around to view it from various different orientations.

`scaling=constrained` This is an option which you include inside the parentheses when using any of the plotting commands listed above. It ensures that the length scales used on all axes are the same, so your graphs are not misshapen. Another useful option is:

`axes=normal` Forces Maple to draw axes.

```
plot3d([x,x,z],x=-2..2,z=-2..2,scaling=constrained,
axes=normal);
```
Plots the plane x=y. Again, it is helpful to view this plot from several different angles. We use this form of the plot statement because we cannot write the equation of this plane in the form z=$f(x,y)$.

```
plot3d(sqrt(1-x^2-y^2),x=-1..1,y=1..1,scaling=
constrained,style=patch);
```
Plots $z=\sqrt{1-x^2-y^2}$. This is the upper hemisphere of the unit sphere.

```
sphereplot(1,theta=0..2*Pi,phi=0..Pi,scaling=
constrained,  style=patch);
```
Plots the unit sphere, `r=1` in spherical coordinates. The `style=patch` option is of course just that; optional. You may prefer the default style, which is `style=hidden`.

```
sphereplot([rho,theta,Pi/4],rho=0..2,theta=0..2*Pi,
scaling=constrained);
```
Plots the cone ϕ=π/4 in spherical coordinates We

use this form of the statement because the equation of this graph is not of the form $\rho=f(\theta,\phi)$.

Finally, similar commands allow you to plot using cylindrical coordinates:

`cylinderplot(expr,theta=0..2*Pi,z=a..b);` Plots graphs of equations of form $r=f(\theta,z)=$`expr`, where (r,θ,ϕ) are cylindrical coordinates.

`cylinderplot([expr1,expr2,expr3],s=a..b,t=c..d);` Plots objects defined by the component equations $r=f_1(s,t)=$`expr1`, $\theta=f_2(s,t)=$`expr2`, and $z=f_3(s,t)=$`expr3`.

Exercise 1: Find the ϕ-coordinates of the circles of intersection of the unit sphere with the cylinder whose equation in cylindrical coordinates is $r=1/2$. Use this information to have Maple plot the region remaining when a hole of radius 1/2 is drilled along the vertical axis of a unit sphere. Submit a copy of this sketch.

Exercise 2: Describe with the aid of a graph the surfaces obeying the following equations:

a) $x+y+z=0$.

b) $\theta=\pi/3$ (in spherical coordinates).

c) $z=2r$ (in cylindrical coordinates).

d) $\rho\sin\phi=4$ (in spherical coordinates. Hint: Depending on the method you choose to graph this, you may wish not to plot points close to $\phi=0$ and $\phi=\pi$.)

Exercise 3: Consider the portion P of the sphere $x^2+y^2+z^2-2z=0$ which lies above the cone $z=\sqrt{x^2+y^2}$. Convert both these equations to spherical coordinates. Use this information and Maple's `sphereplot()` routine to plot P. Submit the plot. What were the restrictions on the coordinates (ρ,θ,ϕ) that you used in order to produce this plot (one way to answer this question is to submit the actual `sphereplot()` statement that you used to create the plot).