Calculus Lab 30—Modelling Internet Growth

Objective: To use mathematical modelling to predict the growth of internet access.

Maple Commands:

dsolve(expr,y(x)); Invokes Maple's **d**ifferential equation **solve**r to solve `expr` for *y(x)*. To use this, first define `expr` to be some ordinary differential equation.

diff(expr,x); Differentiates the expression `expr` with respect to *x*.

plot(expr,x=a..b); Ordinary `plot()` command. You can also specify range `y=c..d`.

plot({expr1,expr2},x=a..b); Plots both `expr1` and `expr2` on one set of axes.

implicitplot(expr,x=a..b,y=c..d); Plots *y* vs. *x*, where *y* is implicitly defined via the expression `expr`. For example, to plot a unit circle, type `implicitplot(x^2+y^2=1,x=-1..1,y=-1..1);`

In this lab, we will use Maple's differential equation solver to solve an equation for us, so we can concentrate on the results. But first we must learn to use the solver, called `dsolve`. Let's try it out on the differential equation

$$\frac{1}{x^2}\frac{dy}{dx} = e^y .$$

ODE:= (1/x^2)*diff(y(x),x)=x*exp(y); Here we create the equation, store it in memory, and call it `ODE`.

dsolve(ODE,y(x)); Here we apply the differential equation solver.

Notice you get a constant of integration. Maple usually employs an underscore character to denote such constants, so the constant may appear as `_C1`, but can take other forms. Notice also that the solution is probably in *implicit* form (*y* isn't isolated; in fact, Maple often isolates the constant of integration instead). Sometimes we can fix that with

solve(%,y(x)); Here `%` refers of course to the last expression.

We could avoid this last problem by using a variation of the `dsolve()` command:

dsolve(ODE,y(x),'explicit'); Solves `ODE` and explicitly isolates *y(x)*.

However, Maple cannot always isolate y. If we use this last form of the `dsolve` command, then Maple will not return any solution when it is unable to isolate y, even when it finds perfectly reasonable solutions in implicit form.

You should be able to solve the above equation without Maple's help because we easily recognize that it is separable. Maple saw this too. To confirm that it did, set

```
infolevel[dsolve]:=1:
```

and repeat the above steps. Notice when you apply `dsolve()` that Maple now tells us *how* it solved the equation: specifically, it says that it used the fact that the equation is separable.

Here's another equation. Try it on your own and see what happens:

$$\frac{dy}{dx} - 2xy = \sin(x)$$

Want Maple to give more information? Try `infolevel[dsolve]:=2:` To turn this feature off, use `infolevel[dsolve]:=0:` By the way, you can apply `infolevel` to extract information when using any Maple command.

Exercise: How fast do people catch up to technology? Let $p(t)$ be the percentage of adults in North America who have internet access. In July 1996, the Boston Globe newspaper reported that 7.7% of adults had access via home, work, or school, but this was increasing dramatically as a function of the time t. Reasonable estimates suggested that it would double every two years, though some expected it to double yearly or even more rapidly. The volume of traffic carried by the internet was doubling every 3 months.

It is believed that dp/dt is proportional to the product of (i) the present percentage $p(t)$ of people already using the internet (since they tell their friends) and (ii) the number $(100\%-p(t))$ of people who do not use it (since new internet users, known as newbies, have to come from the pool of present non-users), so

$$\frac{dp}{dt} = kp(100 - p) \qquad (*)$$

where k is a constant of proportionality. Equation (*) is called the *logistic equation*.

a) Find the general solution of this equation (you may use Maple's `dsolve` here or hand calculations).

b) Let $t=0$ be July 1996. Use the fact that 7.7% of adults were connected to the internet that month to fix the constant of integration in your solution (round off your constant to the nearest whole number).

c) If the percentage of adults who have internet access doubles yearly, then the constant k takes the approximate value k=0.0076 (if t is measured in years), while a doubling time of two years corresponds to k=0.0038. For each of these two possibilities, write down the equation for $p(t)$. There should be no undetermined constants in your equations. Graph these two equations on the same set of axes, using a 20 year period.

d) In what calendar year do you expect 25% of adults in North America to have internet access in each of these two scenarios? When will 90% of adults to have internet access?

e) A more realisitc model might suggest that the maximum percentage of North American adults to have internet access, at least for the foreseeable future, is perhaps 50% of the total population, since economics will prevent some people from ever owning a home computer and since certain occupations do not require computer access at work. How would you revise equation (*) in light of this model? Solve the revised equation (*) and graph the solution, assuming k=0.0038 (a two-year doubling time). In this model, when will 25% of adults have access?

Optional:

f) In part (c), we said that a doubling time for internet access of 2 years resulted in a value for k of k=0.0038. Prove this. [Hint: Use the formula for $p(t)$ from part (b) and the fact that $p(1)=2p(0)$ if 1 year is the doubling time.]

g) Can you find the general solution for equation (*) without using Maple?