Calculus Lab 3—Trigonometry Review

Objective: To recall basic trigonometry from the graphical viewpoint.

Maple Commands:

`expr:=3*x^2+4;` Here we define a Maple expression. The mathematical expression $3x^2+4$ is stored in the Maple variable `expr`. Note the `:=` (not just `=`) and the semi-colon (`;`) terminator.

`plot(expr,x=a..b);` Plots `expr` (a Maple expression) on domain $x \in [a,b]$.

`plot(expr,x=a..b,y=c..d);` Plots `expr` (a Maple expression) on domain $x \in [a,b]$ with range (y-values) restricted to $y \in [c,d]$.

`plot({expr1,expr2},x=a..b);` Plots two (or more) Maple expressions on one set of axes. You can also specify the domain (if your Maple V software is not an earlier version than Release 5).

`cos(x);` Computes cosine of x. The Maple syntaxes for the other basic trig functions are `sin()`, `tan()`, `cot()`, `sec()`, and `csc()`, while π is denoted by `Pi` with a capital `P`. Maple assumes arguments of trig functions are expressed in radian measure.

`solve(eqtn);` Tries to solve a given equation `eqtn`.

`fsolve(eqtn);` Similar to `solve()`, but always returns a floating point decimal solution or approximate solution. If the equation has multiple solutions, `fsolve()` finds only one of them.

We will begin with some basic plots. Try the following:

```
plot(sin(x),x=-4*Pi..4*Pi);
plot(cos(x),x=-4*Pi..4*Pi);
plot(cos(x+2*Pi),x=-4*Pi..4*Pi);
```

These last two plots should look exactly the same. You can make sure by plotting their difference:

```
plot(cos(x+2*Pi)-cos(x),x=-4*Pi..4*Pi);
```

This demonstrates the familiar fact that trig functions are *periodic*—they repeat themselves. The cos and sin functions have period 2π. The tan function has period π, so the following two plots look exactly alike:

```
plot(tan(x),x=-4*Pi..4*Pi,y=-4..4);
```

```
plot(tan(x+Pi),x=-4*Pi..4*Pi,y=-4..4);
```

Here we restrict the range of y-values because the tan function actually diverges to infinity in places, so the y-values get very large. If Maple were to plot such large values, the graph would look distorted.

Exercise 1: Recall that $\sec(x) = \dfrac{1}{\cos(x)}$. Are there any values of x at which $\sec(x)$ is not defined? Is $|\sec(x)|$ ever less than 1? For what x-values between 0 and 2π do you expect $\sec(x)$ ro be an increasing function? a decreasing function? Using the answers to these questions and the graph of $\cos(x)$, try to sketch the graph of $\sec(x)$. Now plot $\cos(x)$ and $\sec(x)$ on a single set of axes (hint: use a restricted range—why?). Pass in a copy of this plot; make sure you identify which curve belongs to which function. Does the result resemble your initial sketch?

Now we will define and plot two new expressions which are the squares of the sine and cosine functions:

`sinSquared:=(sin(x))^2;` Careful with parentheses: we want $(\sin x)^2$, not $\sin(x^2)$.

```
cosSquared:=(cos(x))^2;
plot(sinSquared,x=-4*Pi..4*Pi);
plot(cosSquared,x=-4*Pi..4*Pi);
```

Now let's try something interesting:

```
plot(sinSquared+cosSquared,x=-4*Pi..4*Pi);
```

You should have expected this. What identity is at work here?

If you are good with trig identities, you might even be able to spot the identity at work in the following plot:

```
plot(cosSquared-sinSquared,x=-4*Pi..4*Pi);
```

Exercise 2: Following the above procedure, on separate graphs, plot $\tan^2 x$, $\sec^2 x$, and $\sec^2 x - \tan^2 x$. Hand in copies of these graphs. (Hint: To get decent plots, you should restrict the range of the graphs, maybe to something like `y=-4..4` or perhaps `y=-10..10`) What trig identity is at work in this case? (Hint: If you don't know, make up a plausible identity based on the graph of $\sec^2 x - \tan^2 x$.)

Now let's try to solve an equation involving trig functions. Consider the equation

$$\cos 2x = \cos x \qquad (1)$$

We can submit this to Maple's `solve()` or `fsolve()` function:

```
solve(cos(2*x)=cos(x));
```

Something may be wrong here. Maple may have returned only one or two solutions, but trig functions are periodic—they repeat. Wouldn't we expect the solutions to (1) to repeat also? Let's check.

Exercise 3: Let $f(x) = \cos 2x$ and let $g(x) = \cos x$. Plot both these functions on a single set of axes. Hand in a copy of this plot, clearly identifying which of the two curves belongs to which function. Use the plot to write down all solutions to equation (1). (Hint: Try to see if the solutions recur periodically. It may help to group some solutions into one set and others into another, each set of solutions having a different period. If you state just one solution from each group and also give the period, then this is enough information to specify all the solutions.)

Finally, notice that the slopes of the graphs of the trig functions also repeat periodically. Therefore it is not surprising that the derivatives of trig functions are themselves trig functions. We can determine these derivatives analytically. The derivation requires knowledge of some special limits so instead let's take a graphical approach.

Exercise 4: Consider the family of functions

$$f(x) = \frac{\sin(x+h) - \sin(x)}{h} \tag{2}$$

where h is a definite number (each different h is a different function within this family) and x is the variable. Plot several of these functions, each with a different h-value, on the same set of axes (you don't have to hand in this plot). In particular, try the h-values 2.0, 1.0, 0.5, and 0.1. As h gets smaller, the curve should come to closely resemble the curve of one of the trig functions you've already seen. Which one? What features of the graphs suggested this choice to you?