Calculus Lab 24—Averages

Objective: To understand the average value of a continuous function.

Recall Maple Commands:

`F:=x->3+2*cos(Pi*x);` Defines the function $F(x)=3+2\cos(\pi x)$.

`F(7);` Returns the value of the function $F(x)$ at $x=7$.

`evalf(%);` Evaluate as a floating point (decimal) number (here `%` tells Maple to use the previous expression as the argument for `evalf()`).

`plot(expr,x=a..b);` Plots a graph of `expr` over the domain $[a,b]$.

`plot(expr,x=a..b,y=c..d);` As above, but the range displayed is now restricted to $[c,d]$.

`plot({expr1,expr2},x=a..b);` To plot more than one expression on a single set of axes, enclose the expressions in braces `{}`.

`int(expr,x);` Indefinite integral of `expr` with respect to x.

`int(expr,x=a..b);` Definite integral of `expr` over domain $[a,b]$.

Recall that if f is a continuous function on the interval $[a,b]$ then the average value of f on this interval is given by the formula

$$f_{\text{ave}} = \frac{\int_a^b f(x)dx}{b-a} \tag{1}$$

Exercise 1: Consider that the temperature in Edmonton on a given 24 hour period last summer was given by the formula:

$$T(t) = 16 + 9\sin\left(\frac{\pi t - 8\pi}{12}\right) \tag{2}$$

where t is measured in hours, with $t=0$ being midnight.

a) What is the coldest termperature occurring in that 24 hour period, and at what time does it occur?

b) Compute the termperature every hour on the hour from 3pm ($t=15$) to 9pm ($t=21$), keeping 5 digits of accuracy. Report the average of these 7 discrete values. [Hint: It may help to use Maple to define T as a function (see above) first; then you can sum up values of this function.]

c) Use the formula (1) for the average of a continuous function to compute the average value of the temperature function (2) over the same time interval as in part (b). Call this average value T_{ave}.

d) On a single set of axes, plot equation (2) and the horizontal line $y=T_{ave}$, using the same t interval as you did in parts (b) and (c). Examine the regions between these two curves. What can you observe about the area of these regions? Does the shape of this curve give any hint concerning the origin of the slight discrepency that may exist between the average you found in part (b) and the average you found in part (c)?

Your TA will assign you one or more of the following questions:

2. Plot each of the following functions on the given intervals. Find the largest and the smallest value of the curve on the interval and average the two. Next, take five equally spaced intervals and average these. Lastly, compute the average over the interval using equation (1). Arguing from the shapes of the curves, can you explain any discrepencies between these different techniques?

   a) $g(x)=1/x$ on the interval $[1,5]$.

   b) $f(x)=\sin^4(x)$ on the interval $[0,\pi/2]$.

3. In this question,we discuss whether the concept of an average of a continuous function really makes sense:

   a) The *high temperature* for a given day is the maximum value of the temperature on that day. State in words how you would measure the average high temperature for a year in Edmonton. Would you use formula (1) or another technique?

   b) By way of contrast, the *average annual temperature* for a year should use all values of temperature throughout the year, not just the dialy high temperatures. How do you think the Weather Service measures average annual temperature in practice? Do you think they use formula (1)?

   c) Formula (1) is applicable to the calculation of the average rate of speed of a car during a journey or a race. In this case, what does the numerator of formula (1) represent? What does the denominator represent?