Calculus Lab 23—Integrals of Rational Functions

Objectives: To compute integrals of rational functions using partial fractions decompositions.

Some Maple Commands:

`convert(expr,parfrac,x);` Finds the partial fractions decomposition of expr in terms of the variable x.

`int(expr,x);` Indefinite integral of expr with respect to x.

Finding the partial fractions decomposition of a function is a straightforward exercise, but it can be a very tedious one, so let's make Maple do it for us. To find the partial fractions decomposition of $\dfrac{1}{x^3 - 3x^2 + 4}$.

`z:=1/(x^3-3*x^2+4);`

`convert(z,parfrac,x);`

If you try to do this decomposition by hand, it should not pose too difficult a problem, but it would take you quite awhile to complete.

Exercise 1: Now that Maple has done the decomposition, can you now integrate the resulting expression by hand? You can have Maple integrate $\dfrac{1}{x^3 - 3x^2 + 4}$ directly to check your result.

Similarly, Maple will automatically do the long division necessary to compute the partial fractions decomposition when the numerator is a polynomial of higher degree than the denominator. For example, try to decompose

$$z = \frac{x^3 + 3x^2 - x + 1}{x^2 + 6x + 8}.$$

`z:=(x^3+3*x^2-x+1)/(x^2+6*x+8);`

`convert(z,parfrac,x);`

We can easily integrate this by hand.

Exercise 2: For each of the following functions $f(x)$, Use Maple to find the partial fractions decomposition. Using this decomposition, compute $\int f(x)\,dx$ by hand, showing your working (you need the practice). [Hint: You can easily check your answers by having Maple do the integrals too.]

a) $f(x) = \dfrac{x+3}{2x^2 - 3x - 2}$

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b) $f(x) = \dfrac{x}{x^3 - 2x^2 + x - 2}$

c) $f(x) = \dfrac{x^4 + 1}{x^3 + 1}$

d) $f(x) = \dfrac{[(\sin x) - 2]\cos x}{\sin^2 x + 4\sin x + 3}$ [Hint: First find a substitution to eliminate the trig functions, and then use partial fractions.]

e) $f(x) = \dfrac{\sin x}{\cos^3 x + \cos x}$