Calculus Lab 22—Integration by Parts

Objectives: To practise integration by parts, and apply it to derive a reduction formula.

Some Maple Commands:

`with(student):` Loads the student package .Do this now. We will need this package.

`int(expr,x);` Indefinite integral of `expr` with respect to x.

`Int(expr,x);` Represents the integral in memory, but does not perform the integration.

`intparts(expr,u);` Integrates `expr` by parts. For the second argument, supply the expression that you would like Maple to use to play the role of u in the parts formula.

Recall the integration by parts formula for indefinite integration:

$$\int u\,dv = uv - \int v\,du$$

The trick to successful use of this formula is to know what to choose for u. Once you've chosen u, then dv is also determined, which in turn determines v, so there are no other choices to be made. For example, consider

$$\int x \sin x\,dx$$

There are two choices for u that suggest themselves, u=sin(x) (so $dv=xdx$) or $u=x$ (so dv=sin(x)dx). Let's try them both and see which is best. You can do this by hand, but you can also use Maple:

`with(student):` If you didn't already type this command, do it now.

`z=:Int(x*sin(x),x);` Use capital `I` in the `Int` command, so that Maple doesn't do the integral that would be easier, but we wouldn't learn anything.

`intparts(z,sin(x));` This tells Maple to use integration by parts to compute the integral z, using u=sin(x). Is the integral on the right-hand side simpler or more complicated than the original integral?

`intparts(z,x);` This time, we try using $u=x$ instead. Now is the right-hand side simpler? Can you complete the computation now by hand?

Exercise 1: For each of the following integrals, make two different choices of u. For each choice, apply the integration by parts formula once and report the result. Which choice for u is best in each case, or do both choices work well? Using your preferred choice for u, complete the computation of the integral. You may have to apply the parts formula several times; if so, show each step, including each choice of u. You may perform these calculations using the Maple technique outlined above or hand calculations or a combination of both.

a) $\int x^2 \cos x \, dx$

b) $\int x^2 e^{2x} \, dx$

c) $\int e^x \sin x \, dx$

For integrands of the form x^n times a trig, exponential, or logarithmic function, it is possible to use integration by parts to develop a *reduction formula* that allows you to do the integral simply by plugging the value of n into a formula. Let's now try to develop such a formula.

Have Maple compute $\int x^n \ln x \, dx$ for some values of n:

`int(x*ln(x),x);` This asks Maple to compute $\int x \ln x \, dx$, so this is the case of $n=1$. Notice we are now not telling Maple specifically to use parts to do this integral (although it is probably deciding on its own to use parts for this integral).

`int((x^2)*ln(x),x);` This is the $n=2$ case.

`int((x^3)*ln(x),x);` This is the $n=3$ case.

See a pattern? Can you guess a formula for $\int x^n \ln x \, dx$ when n is completely general?

Exercise 2: Use integration by parts with $u=\ln(x)$ to derive a formula for $\int x^n \ln x \, dx$. Check that the formula agrees with the above examples when $n=1,2,3$. The formula fails when $n=-1$. Can you explain what step in your derivation fails when $n=-1$? Is there any other value of n for which the formula should fail?