Calculus Lab 21—Cylindrical Shells

Objective: To solve an applied problem concerning volumes by the method of cylindrical shells.

Maple Commands:

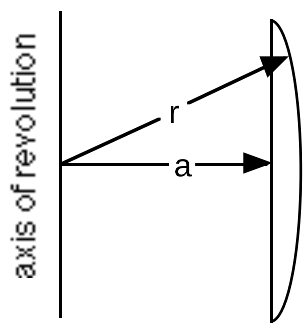
`plot(expr1,x=a..b);` Plots `expr1` on domain [*a*,*b*].

`plot({expr1,expr2},x=a..b);` Plots `expr1` and `expr2` on same set of axes.

`int(expr1,x=a..b);` Definite integral of `expr1` over domain of integration [*a*,*b*].

The following problem is adapted from *Calculus Using Maple* by Edwards and Penney (Prentice Hall, Englewood Cliffs, New Jersey, 1994). For most of the problem, use hand calculations. Use the computer to assist you in solving the cubic equation that appears in Question 2.

A manufacturer of gold wedding rings creates a mold by taking a hollow sphere of radius a and placing inside it a cylinder of radius r. Melted gold is poured into the region between the cylinder and the sphere to form the ring.



Exercise 1:

(i) Use the method of cylindrical shells to show that the volume of this ring is

$$V_1 = \frac{4\pi}{3}\left(r^2 - a^2\right)^{3/2} \qquad (1)$$

(ii) Use the Pythagoras theorem to relate a, h, and r. Using this relationship, re-express equation (1) in terms of h alone (eliminate r and a). Now notice that the volume (and cost) of such a ring does not depend on the radius a of the finger wearing it. Does this surprise you? Can you justify this result in words?

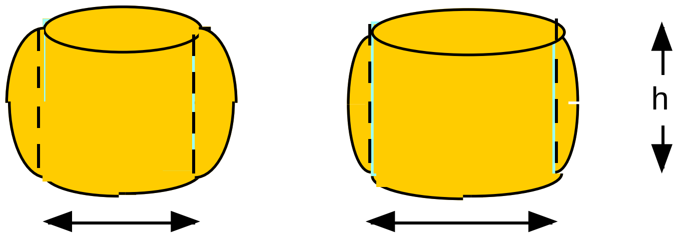
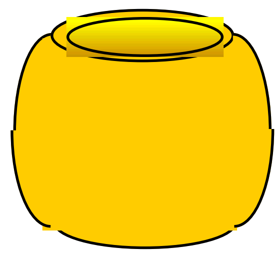
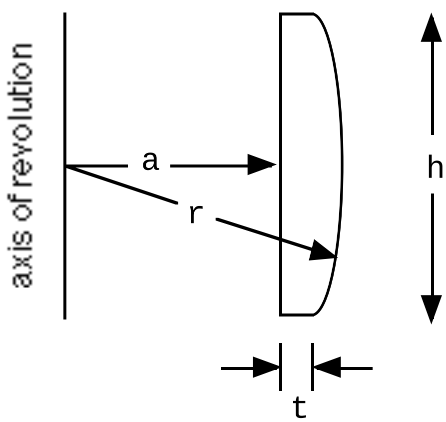


Figure: Rings designed for two different thicknesses of finger, using this method.

The ring desgined above would be rather uncomfortable to wear, since it would have sharp edges. A better design has a lip of thickness t at top and bottom. This new design is shown on the right. It is created by revolving the region depicted below about the vertical axis.





Exercise 2:

(i) Show that the volume of this ring is

$$V_2 = \pi h \left(\frac{1}{6} h^2 + 2at + t^2 \right) \quad (2)$$

[Hint: If you use equation (1) and if you know that the volume of a cylinder of radius R and height H is $\pi R^2 H$, you can do this without any calculus. By the way, the volume of the ring in this case does depend on the thichness of the finger, but only slightly.]

(ii) You can easily measure the circumference of your ring finger using a piece of string and a ruler. If you have a ruler and string handy, make this measurement; otherwise assume the circumference to be 63mm (that's the circumference of the ring finger of the author of this lab). Assume a jeweller will charge \$35/g for the gold that goes into the ring and that the gold has density 18.4g/cm^3. If you are willing to pay \$500 for your ring, how big a ring can you afford; i.e. find h to the nearest mm using equation (2). Remember not to confuse the *circumference* of your ring finger with the radius needed in equation (2).

[Hint: Equation (2) is cubic in h and therefore you are not likely to be able to isolate h. Instead, can you think of a method to find h approximately by using a graph?]