Calculus Lab 20—Volumes of Solids of Revolution

Objective: To practise the formulas for volumes of solids of revolution.

Recall Maple Commands:

`plot(expr,x=a..b);` Plots a graph of `expr` over the domain $[a,b]$.

`plot(expr,x=a..b,y=c..d);` As above, but the range displayed is now restricted to $[c,d]$.

`plot({expr1,expr2},x=a..b);` To plot more than one expression on a single set of axes, enclose the expressions in braces `{}`.

`int(expr,x);` Indefinite integral of `expr` with respect to x.

`int(expr,x=a..b);` Definite integral of `expr` over domain $[a,b]$.

`evalf(%);` Evaluate as a floating point (decimal) number (here `%` tells Maple to use the previous expression as the argument for `evalf`).

Recall some of the formulas for computing volumes of solids of revolution:

$V = \int_a^b \pi[f(x)]^2\,dx$ gives the volume obtained by rotating about the x-axis the region between the x-axis and the graph of $f(x)$, using the method of disks.

$V = \int_a^b 2\pi x f(x)\,dx$ gives the volume obtained by rotating the same region about the y-axis, using the method of cylindrical shells.

Exercise: For each of the following examples, sketch the specifed region R (Maple may help you draw the necessary graphs). Revolve R about the indicated axis to create a solid of revolution and sketch this solid. Set up the formula for the volume of this solid and compute the volume.

a) $f(x)=(x^2+1)^{1/2}$, $1\le x\le 2$. R is the region between the x-axis and the graph of $f(x)$. Revolve R about the y-axis. Compute the volume of this solid by hand using cylindrical shells.

b) $f(x)=(x^2+1)^{1/2}$, $1\le x\le 2$. This time, R is the region between this graph and the y-axis. Revolve R about the y-axis and compute the volume of the resulting solid (using Maple or by hand) using the method of disks.

c) $f(x)=x\sin(x)$, $0\le x\le\pi$. R is the region between the x-axis and the graph of $f(x)$. Revolve R about the x-axis, using disks. You will need Maple to compute the volume integral. (If you prefer, you can convert the answer, likely given in terms of π, to a 10-digit decimal approximation using the `evalf()` command.)

d) $f(x)=x^2, g(x)=x^{1/2}$. R is the region between these curves. Rotate R about the x-axis. Compute the volume by any method you find convenient (shells or disks, Maple or hand calculations).