

Calculus Lab 2—Functions and Expressions

Reminder: Pass in the answers to the *Exercises*. Copy any necessary results from the screen onto the paper you pass in.

Objectives: We will learn about the distinction between expressions and functions in Maple. Using function notation, we will examine inequalities, limits, and (optionally) piecewise-defined functions, emphasizing graphical techniques.

1. Functions and Expressions:

Define the following *expression*:

```
e1:=x^3+3*x;
```

To evaluate it at $x = 1/2$, we use the `subs()` command (short for ‘substitute’):

```
subs(x=1/2,e1);
```

To get a decimal answer, we must use `evalf()`:

```
evalf(%);
```

By way of contrast, Maple defines *functions* by using the arrow notation. To make it, you first type a dash `-` and then follow it with a ‘greater than’ symbol `>` to get `->`. Define the following function:

```
f:= x -> x^3+3*x;
```

This notation says that f is the function that takes x as input, computes $x^3 + 3x$, and returns the result as output. To evaluate a function, we *do not need the `subs()` command*. Type:

```
f(1/2);
```

2. Plotting functions and solving inequalities:

The plotting commands for functions are slightly different than those for expressions. We practise them by using plots to help solve the inequality $x > x^2$. By this, we mean we will find all x -values that are greater than their own squares. Our technique is to treat each side of the inequality as a function, say $f(x) = x$ and $g(x) = x^2$. If we plot both functions on one set of axes, then we can look for those x -values where the graph of $f(x)$ is higher than the graph of $g(x)$. Those x -values are the solutions.

```
f:=x->x;
```

```
g:=x->x^2;
```

```
plot({f,g});
```

Since we did not specify a domain and range, Maple used its default domain here. The resulting plot is not very useful.

`plot({f,g},-2..2);` Now that's better. Here we've specified the domain $x \in [-2,2]$.

`plot({f,g},-2..2,-3..3);` Even better. This time we've also specified the range $y \in [-3,3]$.

For which x -values does the graph of f lie above that of g ?

Notice the command syntax: We simply said `-2..2` instead of `x=-2..2` when specifying the domain, and we also left off the `y=` when specifying the range. That's the basic difference between functions and expressions: functions already know what the independent variable is (x , or some other symbol), so they know that when we type `-2..2` in the `plot()` command, we are referring to x . Expressions don't know this and need to be told.

Exercise 1: Use a graph to find all x such that the inequality $5x^3 < 8x - 1$ is true. Sketch the graph and indicate on the graph the region where the inequality is true. What would be the difference in the solution if the $<$ sign in the inequality were replaced with \leq ?

3. Limits: Try the following:

```
f:=x -> (x^2-4)/(x-2);  
f(2);
```

Why does Maple complain? Plot this function. Choose the domain so that you zoom in very closely on $x=2$. Is anything wrong with this graph? Now try

```
limit((x^2-4)/(x-2),x=2);
```

This is an example of a function with a removable discontinuity.

Exercise 2: The sine and cosine functions in Maple are denoted by `sin(x)` and `cos(x)` respectively. Use Maple to evaluate the limits of $\sin(x)/x$ and $(1-\cos(x))/x$ as $x \rightarrow 0$ by three different methods (notice these functions are undefined at $x=0$). First, evaluate each of these two expressions at $x=0.1$, $x=0.01$, $x=0.001$, and $x=0.0001$, and record the results in a table. Use these values to guess the limits. Next, use Maple's limit command to evaluate the limits exactly. Finally, use Maple to plot both these functions and read the limits off the plots. Copy the plots onto paper and submit them with your answer.

Lastly, Maple can compute left-hand and right-hand limits. For example, the function `abs(x)` is used by Maple to compute the absolute value of x , denoted in standard mathematical notation by $|x|$. Plot $|x|$.

```
plot(abs(x),x=-2..2);
```

Now divide this function by x to get a new function $|x|/x$. Compute the left-hand and right-hand limits of the function $|x|/x$ as $x \rightarrow 0$. The necessary Maple commands are

```
limit(abs(x)/x,x=0,left);    and
limit(abs(x)/x,x=0,right);
```

Plot the function $|x|/x$ using Maple.

```
plot(abs(x)/x,x=-2..2);
```

Exercise 3: Plot the functions $1-x^2$ and $|1-x^2|$ on a single set of axes. Hand in a copy of this plot, labelling each curve with the function that produces it.

Now define the new function $f(x) = \frac{|1-x^2|}{1-x}$ and compute its limits as $x \rightarrow 1$ from the left and as $x \rightarrow 1$ from the right. Provide a plot of $f(x)$ illustrating the behaviour as $x \rightarrow 1$ (it may improve the plot if you specify `discont=true` as an option in the Maple `plot()` command). What happens when you try to compute the limit at $x=1$ without specifying a direction (left or right)? Why?

Optional:

4. Piecewise-defined Functions:

We now define the function:

$$g(x) = \begin{cases} x-1, & x \leq 2 \\ 5-4(x-3)^2, & 2 < x < 4. \\ x+1 & x \geq 4 \end{cases}$$

This function is very hard to understand without a picture, so we will have Maple plot it just as soon as we define it. The key point is that the function is given by different expressions for different parts of the domain. It is therefore piecewise-defined.

```
g:=x -> piecewise(x<=2, x-1, x<4, 5-4*(x-3)^2, x+1);
plot(g,-2..8);
```

Exercise 4: Plot the piecewise-defined function $h(x) = \begin{cases} -x^2, & x < -1 \\ x, & -1 \leq x \leq 1. \\ -x^2, & x \geq 1 \end{cases}$