Calculus Lab 12—Difference Quotients and Derivatives

Objective: To compute difference quotients and derivatives of expressions and functions.

Recall Plotting Commands:

`plot({expr1,expr2},x=a..b);` Plots two Maple expressions on one set of axes.

`plot({f,g},a..b);` Plots two Maple functions on one set of axes.

`plot({f(x),g(x)},x=a..b);` This allows us to plot the Maple functions `f` and `g` using the form of `plot()` command apprpriate to Maple expressions. If `f` and `g` are Maple functions, then `f(x)` and `g(x)` are the corresponding Maple expressions. The output of this `plot()` command is precisely the same as that of the preceding (function version) `plot()` command.

---

1. We begin by using Maple to compute difference quotients and, from them, derivatives. Try the following sequence of commands:

   `f:=x->1/(x^2-2*x+2);` This defines the function $f(x)=\dfrac{1}{x^2-2x+2}$.

   `(f(2+h)-f(2))/h;` This is the *difference quotient* of f at the point $x = 2$.

   `simplify(%);` Simplifies the last expression.

   `limit(%,h=0);` This gives the derivative of f at the point where $x = 2$.

Exercise 1: Find the difference quotient and derivative of this function at a general point x (hint: make a simple modification of the above steps). Use this to evaluate the derivative at the points $x=-1$ and $x = 4$. (It may help to remember the `subs()` command here; for example, `subs(x=1,e1);` means substitute $x = 1$ into the expression `e1`). Plot f and its derivative on the same set of axes, using an appropriate domain; copy down the graph and hand it in for marking. At what value of x is the derivative equal to zero? For what value of x does the tangent line to f appear to have zero slope?

Plotting Hint: You may encounter difficulty plotting both f and its derivative using one command, since f is a function but the derivative is a Maple expression, and the `plot()` command treats these differently. The easiest thing to do is to plot them both as expressions. See the discussion of the `plot()` commands at the top of the page.

2. You have just computed the derivative of the function $f(x)=\dfrac{1}{x^2-2x+2}$. However, Maple can also do this in one step using the *D* command. *D* differentiates functions, not expressions.

```
f:=x->1/(x^2-2*x+2);
```

`fprime:=D(f);` This defines `fprime` to be the derivative at any x.

`fprime(4);` This is the derivative at $x = 4$.

Compare the output from these commands to your answers in the previous exercise.

> *Exercise 2:* Define the function $f(x)=\dfrac{|1-x^2|}{1-x}$. Plot it and its derivative on the same set of axes. From the graph, read off the slope of the tangent line to the graph of $f(x)$ as you approach $x=1$, first approaching from the left and then from the right. Can you define a tangent line exactly at $x=1$? Using the graph of the derivative of $f(x)$, decide what are the limits of this derivative as you approach $x=1$ from the left and from the right. Does the derivative have a value at $x=1$?

3. We can take the derivative of a mere expression, as opposed to a function (remember that Maple treats functions and expressions differently). To do this, instead of using `D`, we must use the `diff` command:

```
diff(1/(x^2-2*x+2),x);
```

We could also have said

```
expr1:=1/(x^2-2*x+2);
diff(expr1,x);
```

Now notice we must include an x as a second argument to remind Maple that the derivative is taken with respect to x. (This is reminiscent of the `plot()` command; when plotting expressions, we must say "x =" when specifying the domain.) For a function, this would not be necessary since the "`x->`" notation that we use when defining a Maple function makes it clear that x is the variable on which the function depends.

> *Exercise 3:* Plot $y=x^4-3x^3+\dfrac{1}{2}x-3$ (don't copy down this plot—we'll do another below). Where does the tangent line have slope equal to zero? It is rather hard to tell by inspection, so instead use the following method:
>
> First compute the derivative of y and call the resulting expression `yprime`. We will apply a new command, `fsolve()`, to this derivative. Recall that the

statement `solve(expr=0);` will find zeroes of the expression `expr.` You can apply the `solve()` command to the derivative of y, but the response may not be very helpful. Instead, apply the new command

```
fsolve(yprime=0);
```

which means "solve `yprime=0`, and report the answers as floating point (decimal) numbers, to 10 decimal places of accuracy."

Now plot both y and its derivative on a single set of axes (be careful to choose a useful domain and range). Which is which? Use information from the graph of the derivative to indicate those points where the tangent to the graph of y has zero slope.

Compute the derivative of the derivative (commonly called the second derivative) of y.