Calculus Lab 11—Slope of the Tangent Line

Objectives: To understand the slope of the tangent line as a limiting procedure.

Recall:

`expr1:=x^2;` Defines `expr1` to represent the expression x^2.

`plot(expr1,x=-2..2);` Plots `expr1` on the interval from $x=-2$ to $x=2$.

`plot({expr1,expr2,expr3},x=-2..2);` Plots many expressions on one set of axes.

We will need special Maple commands that belong to the student package of Maple and are not loaded into memory when the program starts. To load these commands, type:

```
with(student);
```

Now recall the *point slope form* of the equation of a line. If (x_0,y_0) is a known point on a line of slope m and if (x,y) also lies on the line, then (x,y) obeys:

$$(y-y_0)=m(x-x_0)$$

Example: If (1,2) and (3,-2) are two points on a straight line, what is the point-slope form of the equation of this line?

To answer this, we must know m. Since we are given two points, we can determine this using the `slope` command in the student package of Maple. We supply this command with the two given points, enclosed in square brackets:

```
slope([1,2],[3,-2]);
```

Maple returns the answer -2. (If this doesn't work when you try it, you may have forgotten to execute the `with(student);` command. Do it now and try again.) We plug this result into the point-slope equation above, and use either of our given points as the point (x_0,y_0). The answer is therefore either

$$(y-2)=-2(x-1) \qquad \text{or} \qquad (y+2)=-2(x-3).$$

Can you see why both these answers are equivalent?

Exercise 1: Find the point-slope form of the line passing through (-4,-2) and (1,3/2). Plot this line. Copy the plot onto your answer sheet, correctly indicating these points and the point where this line crosses the y-axis.

Reminder: In what follows, you may sometimes wish to ask Maple to give you the exact coordinates of a point on a plot on your computer screen. To do this, position the cursor in the plot area and click once. This selects the plot (you'll

see the border of the plot area change). Then place the cursor exactly over the point whose coordinates you want to know. Click again. The coordinates will now be displayed in an area near the upper left corner of your screen (on the Maple menu bar). If you have told Maple to display the plot in its own separate window (this option is available), the coordinates may be displayed instead at the top of the plot window. Make sure you click twice, once to select the plot and once more to get the coordinates of the point, otherwise the coordinates being displayed may be those from a previous operation.

Exercise 2: Define $y = 1/x^2$ and consider its graph. Notice that $(1,1)$ lies on this graph. What is the y-value of the point on this graph with $x = 1.2$? (Hint: Define `y:=1/x^2;` and use the substitute command `subs(x=1.2,y);`). Use the `slope` command to determine the slope of the line passing through this point and the point $(1,1)$. Repeat this exercise, using instead the value $x = 1.1$ to find a new point on the curve. Again find the slope of the line through this new point and the point $(1,1)$. Lastly, repeat once more with $x = 1.01$. Record these slopes to hand in.

Exercise 3: Repeat Exercise 2 to get three more lines. Again, use $(1,1)$ as one point on each line, but for the other point first use $x = 0.8$, together with the corresponding y-value obtained from the curve $y = 1/x^2$. Then repeat with $x = 0.9$ and, lastly, use $x = 0.99$. What value do the slopes (from both exercises) appear to be approaching as the second point moves closer to $(1,1)$?

Plot the lines obtained in this exercise and the curve $y = 1/x^2$ all on the same set of axes. Be sure to choose the domain so that the lines can clearly be seen. [Hint: The domain *should not include* $x = 0$. Why not?] Copy this plot onto paper to be passed in.

Exercise 4: The Maple student package command

```
showtangent(y,x=1,x=0.5..2);
```

will draw a plot of the expression y on the domain $0.5 \le x \le 2$, and will plot on the same set of axes the line tangent to this curve at $x = 1$. Use this command to make this plot for $y = 1/x^2$ and copy it onto paper to be handed in. From the plot, read off the slope of the tangent line. How does this number compare to the slopes obtained in Exercises 2 and 3?