Banach lattice for distributional integrals

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Integrals defined by their primitives

**Lebesgue**

\[ f \in L^1(\mathbb{R}) \quad \int_a^b f = F(b) - F(a) \quad F \in AC \]

\[ F'(x) = f(x) \text{ a.e.} \]

**Henstock-Kurzweil**

\[ f \in HK \quad \int_a^b f = F(b) - F(a) \quad F \in ACG_* \]

\[ F'(x) = f(x) \text{ a.e.} \]

**wide Denjoy**

\[ f \in D \quad \int_a^b f = F(b) - F(a) \quad F \in ACG \]

\[ D_{ap}F(x) = f(x) \text{ a.e.} \]

\[ C^1 \subsetneq AC \subsetneq AC^G_* \subsetneq AC^G \subsetneq C^\circ \]
Distributions

Test functions
\( \mathcal{D} = C_c^\infty(\mathbb{R}) \)

Convergence \( \phi_n \to 0 \) 
There is compact \( K \) such that for all \( n \geq 1 \) 
\( \text{supp}(\phi_n) \subset K \).
For each \( m \geq 0 \), \( \phi_n^{(m)} \to 0 \) uniformly on \( K \) as \( n \to \infty \).