Math 113/114–Final Exam Sample Problems

1. Find dy/dx. Do not simplify your answers.

(a) $y = (1 - x - x^3)\sec^2(x^2 - 1)$ (b) $y = (\tan(3x))^{\ln x}$ (c) $y = \tan^3\sqrt{x^2 - \sin x}$

(d) $y = \dfrac{\sin^3 x\sqrt{3x^5 + 1}}{e^{x^3}(2 - 5x^2)^5}$ (e) $y = \tan\left(x^2 - \sin\sqrt{1 - x^3}\right)^4$ (f) $\sin(x - y^2) = x^2 - xy$

(g) $y = (\sec x)^{\tan x}$ (h) $e^{xy} - y^2 = \ln x$

2. Evaluate the limits.

(a) $\lim\limits_{x\to\infty}(x - \sqrt{x^2 + x + 1})$ (b) $\lim\limits_{x\to 0}\dfrac{1 - \cos x}{x\sin 2x}$ (c) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\tan 2x}$

(d) $\lim\limits_{x\to 2}\dfrac{\sqrt{2x} - x}{x - 2}$ (e) $\lim\limits_{x\to -\infty}\dfrac{2x}{\sqrt{x^2 + 1}}$ (f) $\lim\limits_{x\to -\infty}\dfrac{1 - 4x + x^3}{3x^2 - x + 4}$

3. Evaluate the integrals.

(a) $\displaystyle\int \frac{1 - x^2 - 3x^2}{x^{\frac{2}{3}}}\,dx$ (b) $\displaystyle\int x\sin(1 - 2x^2)\,dx$ (c) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin 2x}{\sqrt{1 + \cos 2x}}\,dx$

(d) $\displaystyle\int_0^2 |3x - 1|\,dx$ (e) $\displaystyle\int_0^1 |2x^2 + x - 1|\,dx$ (f) $\displaystyle\int \sin\theta\cos\theta(2 + \cos^2\theta)^2\,d\theta$

(g) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{e^{\tan 7x}}{\cos^2 7x}\,dx$ (h) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^3\sin^4 x}{1 + x^4}\,dx$ (i) $\displaystyle\int x^5\sqrt{x^3 + 1}\,dx$ (j) $\displaystyle\int x\sqrt{2 - x}\,dx$

(k) $\displaystyle\int \frac{\sec^2 5x}{3 + 7\tan 5x}\,dx$ (l) $\displaystyle\int (1 + e^{\cos x})\sin x\,dx$ (m) $\displaystyle\int_1^e \frac{\cos^2(\pi\ln x)\sin(\pi\ln x)}{x}\,dx$

4. Use the Fundamental Theorem of Calculus to find $f'(x)$.

(a) $f(x) = \displaystyle\int_{\tan x}^2 \sqrt{1 + t^4}\,dt.$ (b) $f(x) = \displaystyle\int_{\cot^2(1 - \tan x)}^0 \cos(t^2)\,dt.$

5. Find the area bounded by the curve $y = x^2 - 2x$ and the x-axis between $x = -1$ and $x = 1$. Sketch the graph and indicate the bounded region.

6. A right circular cylinder is inscribed in a cone with height 20 cm and base radius 5 cm. Find the largest possible volume of such a cylinder.

7. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 16 - x^2$.

8. Find the dimensions of the rectangle of largest area that can be inscribed in the circle with the equation $x^2 + y^2 = r^2$.

9. State the Mean Value Theorem. Given that f is continuous on $[2, 5]$ and $1 \le f'(x) \le 4$ for all $x \in (2, 5)$. Use the Mean Value Theorem to show that $3 \le f(5) - f(2) \le 12$.

10. Use Rolle's Theorem to show that the equation $3x + 2\cos x + 5 = 0$ has exactly one real root.

11. Let $f(x) = 4\cos x$ on $[0, \frac{\pi}{2}]$. Given the sample points $x_i^* = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$. Find the area of the approximating rectangles. Sketch the graph of f and show the approximating rectangles.

12. Evaluate the definite integral of $f(x) = 2 + x^2$ on $[1, 3]$ using the definition (i.e., the limit definition using a Riemann sum). Use right endpoints.

$$\left(\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \right)$$

13. For the following, find y' and y''. Simplify your answers.

(a) $y = \dfrac{x}{(x-1)^2}$ (b) $y = x\sqrt{9 - x^2}$

14. For #13(b), does the curve have an absolute max and/or absolute min? Justify.

15. For #13(a), sketch the graph by finding: domain, critical points, intervals of increase/decrease, local extrema, intervals of concavity, inflection points, asymptotes.

16. The volume of a right circular cylinder is increasing at 2 cm^3/min. The radius is increasing at 1 cm/min. How fast is the height of the cylinder changing at the time when the radius is 5 cm and the volume is 60 cm^3?

17. The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400-ft tall building increasing when the angle of elevation of the sun is $\pi/6$.