MATH 113/114

Midterm Review Questions

Answer Key

On the midterm show all of your work clearly and make sure that your final answer is clearly stated. No calculators, cell phones or formula sheets will be allowed. This review does not reflect the length of your midterm.

(1) Find all x values that satisfy $|3x + 2| < 1$. Give your answer in interval notation. All x in $(-1, -1/3)$.

(2) For each function find the domain and range, and determine if the function is even, odd, or neither.

(a) $f(x) = \sqrt{x^2 - 7}$

$\mathcal{D}_f = (-\infty, -\sqrt{7}] \cup [\sqrt{7}, \infty)$, $\mathcal{R}_f = [0, \infty)$, f is even.

(b) $f(x) = \ln(9 - x^2)$

$\mathcal{D}_f = (-3, 3)$, $\mathcal{R}_f = (-\infty, \ln 9)$, f is even.

(c) $g(x) = x - \sin x$

$\mathcal{D}_g = (-\infty, \infty)$, $\mathcal{R}_g = (-\infty, \infty)$, f is odd.

(3) Find $\sin(\pi/12)$ using the fact that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$.

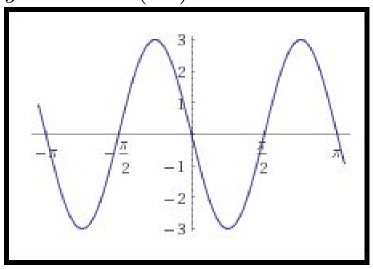
$\sin(\pi/12) = (\sqrt{3} - 1)/(2\sqrt{2})$

(4) If θ is in the interval $[0, \pi/2]$ and $\sin\theta = 1/3$, then what is $\cos\theta$?

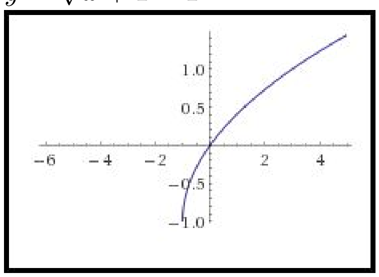
$\cos\theta = \sqrt{8}/3$

(5) Sketch the following two graphs.

(a) $y = -3\sin(2x)$



(b) $y = \sqrt{x+1} - 1$



(6) Calculate $8\log_2(6) - 4\log_2(18)$. 4

(7) For $f(x) = \dfrac{x-1}{2+x}$, find the function $f^{-1}(x)$.

$f^{-1}(x) = (2x+1)/(1-x)$

(8) Find the vertical and horizontal asymptotes of

$$f(x) = \frac{2x^2 + x - 3}{x^2 - 1}\,.$$

VA: $x = -1$, HA: $y = 2$

(9) Find the vertical and horizontal asymptotes of

$$f(x) = \frac{1 - 4e^x}{e^x - 5}\,.$$

VA: $x = \ln 5$, HA: $y = -4$ and $y = -1/5$

(10) Find the following limits if they exist. If the limit does not exist, explain why.

(a) $\lim_{x\to 3} \frac{x^2-x-6}{x^2-2x-3} = 5/4$

(b) $\lim_{x\to 2} \frac{3-\sqrt{x^2+5}}{x-2} = -2/3$

(c) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-\sqrt{2x-4}} = -4$

(d) $\lim_{x\to\infty} \frac{2\cos x+2}{x} = 0$

(e) $\lim_{x\to -\infty} \dfrac{\sin^2 x}{x} = 0$

(f) $\lim_{x\to 1} \frac{x^2-1}{|x-1|}$ DNE

(g) $\lim_{x\to 0} \frac{1-\cos^2 x}{x\sin 3x} = 1/3$

(h) $\lim_{x\to 0} \dfrac{\sin(3x)}{\tan(8x)} = 3/8$

(i) $\lim_{x\to\infty} \frac{3x^3+7}{2x^3+4x^2+2} = 3/2$

(j) $\lim_{x\to -\infty} \dfrac{-\sqrt{4t^2+10}}{5t} = 2/5$

(k) $\lim_{x\to\infty} \dfrac{-\sqrt{4t^2+10}}{5t} = -2/5$

(l) $\lim_{x\to 0} \ln(1-\cos x) = -\infty$

(11) In each question, find a value of c that will make the function continuous on the interval $(-\infty, \infty)$. In each case explain your answer.

(a)

$$f(x) = \left\{ \begin{array}{ll} x^2-1, & x \le 2 \\ 3x+c, & x > 2 \end{array} \right\}$$

$c = -3$

(b)

$$f(x) = \left\{ \begin{array}{ll} \cos x, & x \le \pi \\ x+c, & \pi < x \end{array} \right\}$$

$c = -1-\pi$

(12) Use the intermediate value theorem to prove that the function $f(x) = x^3 - 7x + 2$ has a root between $x = 0$ and $x = 1$. (Be sure to make clear why you can use IVT for this question, i.e. What conditions does f need to satisfy for IVT to be applicable?)

f is a polynomial, so it is continuous on $[0,1]$. $f(0) = 2 > 0$ and $f(1) = -4 < 0$, so by IVT f has a root x in $(0,1)$.

(13) (a) Use the definition of the derivative to find $f'(1)$ when $f(x) = 1/\sqrt{x}$. No points will be given if the definition is not used. $f'(1) = -1/2$

(b) Find the equation of the tangent line of $f(x) = 1/\sqrt{x}$ at $x = 1$. $y = (-1/2)x + (3/2)$

(14) (a) Use the definition of the derivative to find $\frac{d}{dx}(\cos x)$. No points will be given if the definition is not used. $\frac{d}{dx}(\cos x) = -\sin x$

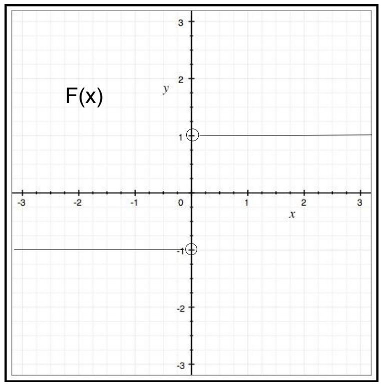
(b) Find the equation of the line tangent to $\cos x$ at $x = -\pi/3$. $y = (\sqrt{3}/2)x + (\sqrt{3}\pi + 3)/6$

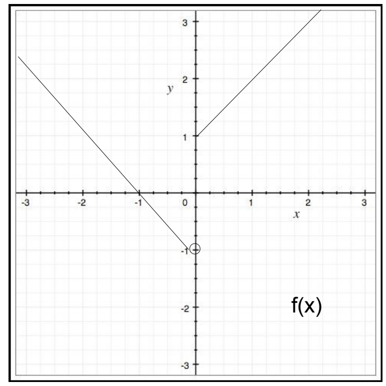
(15) Sketch the function

$$F(x) = \left\{ \begin{array}{cc} -1, & x < 0 \\ 1, & x > 0 \end{array} \right\}$$

and using this, sketch a function that has $F(x)$ as its derivative.

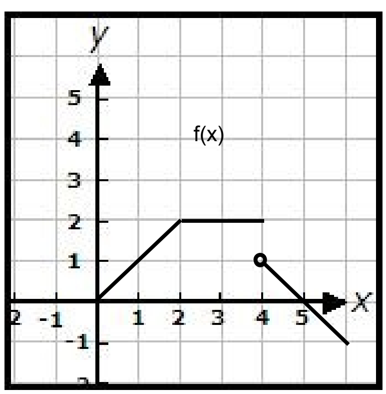
There are many functions $f(x)$ with $f'(x) = F(x)$, below is just one such $f(x)$

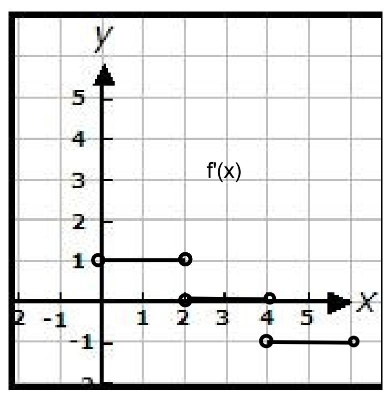




(16) Draw the graph for $y = f(x)$, and separately, the graph of $f'(x)$ given that

$$f(x) = \left\{ \begin{array}{cc} x & 0 \leq x \leq 2 \\ 2 & 2 < x \leq 4 \\ -x+5 & 4 < x \leq 6 \end{array} \right\}$$





(17) Find the derivative of the following functions. You do not need to simplify your answers.

(a) $h(x) = \sec 2x - 5\cot(x-1)$

$h'(x) = 2\sec(2x)\tan(2x) + 5\csc^2(x-1)$

(b) $f(x) = x\cos(2x^2 - 1)$

$f'(x) = \cos(2x^2 - 1) - 4x^2 \sin(2x^2 - 1)$

(c) $g(x) = \frac{x^2}{\sqrt{x - x^2 + 7}}$

$g'(x) = (2x)(x - x^2 + 7)^{-1/2} - \frac{x^2 - 2x^3}{2}(x - x^2 + 7)^{-3/2}$

(d) $f(x) = (6x^3 - 5\cos x)^{10}$

$f'(x) = 10(6x^3 - 5\cos x)^9(18x^2 + 5\sin x)$

(e) $g(x) = a^{\sin x}$

$g'(x) = a^{\sin x}(\ln a)(\cos x)$

(18) Find the following:

(a) dy/dx for $y = \sin(1)\sin^2(x^4) + \sec^2(\pi x^2 + x + 1)$

$dy/dx = 8x^3 \sin(1)\sin(x^4)\cos(x^4)$

$+ (4\pi x + 2)\sec^2(\pi x^2 + x + 1)\tan(\pi x^2 + x + 1)$

(b) $f''(x)$ for $f(x) = \dfrac{x+1}{x-2}$

$f''(x) = \dfrac{6}{(x-2)^3}$

(c) $g^{(3)}(t)$ for $g(t) = t^3 + \cos(10t)$

$g^{(3)}(t) = 6 + 1000\sin(10t)$

(19) At the edge of a cliff, a ball is tossed straight up in the air. At time t the ball is $f(t) = 80 + 64t - 16t^2$ feet from the ground that is below the cliff.

(a) What is the maximum height that the ball reaches? 144 feet

(b) At what speed is the ball traveling when it hits the ground? 96 ft/sec