MATHEMATICS 113/114

MIDTERM EXAMINATION

VERSION 1

DATE: October 22, 2008 TIME: 50 MINUTES

INSTRUCTIONS:

1. **Make sure that the exam has 7 pages including the cover page.**
2. **Books, notes or calculators are not permitted.**
3. **Show all your work to receive full credit.**

#	Value	Mark
1	12	
2	10	
3	10	
4	10	
5	8	
Total	50	

1. (12 points) Find y'. Do not simplify your answers.

   (a) (4 points) $y = \cos^5\left(\dfrac{2x^7 - x}{5 - x^3}\right)$

   (b) (4 points) $y = (\tan(e^{x^2}) - 5x^{\frac{1}{3}})^8\,(3 - e^{\cos x})^{10}$

   (c) (4 points) $y = \sin(\cot^3\sqrt{2 - x^5})$

2. (10 points) Evaluate the following limits. Show steps. If the limit does not exist, explain why not. You will not receive any credit if l'Hospital's rule is used.

   (a) (3 points) $\lim_{x\to 3} \dfrac{x^2 - x - 6}{9x - x^3}$

   (b) (3 points) $\lim_{x\to 0} \left(\dfrac{1}{x\sqrt{1+x}} - \dfrac{1}{x} \right)$

   (c) (4 points) $\lim_{x\to 5} \dfrac{x^2 - 3x - 10}{|5 - x|}$

3. (10 points)

(a) (6 points) Use the definition of the derivative of f as a **function** to find $f'(x)$ where $f(x) = \dfrac{x}{2x-3}$.

(b) (4 points) Find the domain of the function $f(x) = \dfrac{1}{(e^x - 1)(x+1)} + \dfrac{1}{\sqrt{2-x^2}}$ in the interval form.

4. (10 points)

   (a) (4 points) Find the point(s) on the curve $y = \sqrt{8x - \frac{2}{3}x^3}$ at which the tangent line(s) is(are) horizontal, and write down the equation(s) of the tangent line(s).

   (b) (3 points) Find $f'(x)$ and $f'(2)$ if $f(x) = x^3 \sin(g(x))$ where g is a differentiable function, and given that $g(2) = \frac{\pi}{3}$ and $g'(2) = -\frac{1}{4}$.

   (c) (3 points) Evaluate $\lim_{x \to -\infty} \frac{\sqrt{3 + 4x^2}}{2 - 7x}$.

5. (8 points) Let

$$f(x) = \begin{cases} \dfrac{\sin^2 3x}{x^2}, & x < 0 \\ 2a, & x = 0 \\ 4a + 3b - x^2, & x > 0 \end{cases}$$

(a) Write down the precise condition under which the given function f is continuous at $x = 0$.

(b) Find the values of a and b so that the function f is continuous at $x = 0$.