Optional Appendix - Curve Fitting

In a hyperbolic function, there are two unknown parameters, a and b, which appear nonlinearly. The goal of using linear regression is to rewrite the hyperbolic function in a form that contains two parameters that appear linearly, which we can do as follows:

$$\begin{aligned} N &= 1 + \frac{a}{H-b} \\ N - 1 &= \frac{a}{H-b} \\ \frac{1}{N-1} &= \frac{H-b}{a} \\ \frac{1}{N-1} &= \frac{1}{a}H - \frac{b}{a} \end{aligned}$$

The last expression is in the form $y = mx + B$, with

$m = \frac{1}{a}$,

$B = \frac{-b}{a}$,

$y = \frac{1}{N-1}$,

$x = H$

So, if the original data of N_{avg} versus H can be approximated by a hyperbolic function, then the transformed data $\frac{1}{N-1}$ versus H can be approximated by a straight line.

To find the best fit line for this data, calculate the transformed data using the formula above. Then, using your graphing calculator, you can perform a linear regression on the transformed data to find m and b. From m, you can calculate the value for a that we need in equation 9, and from B, you can find the value for b.