MATH 538: Techniques in Applied Mathematics

Assignment 4

Due Tuesday, April 11 by 3pm

1. **(/40)**
   At a linearly-sloping beach the water depth increases as $H = sx$ in which $0 < s \ll 1$ is the (constant) slope. A packet of small-amplitude water waves of wavenumber k_x approaches the beach passing through $(x_0, 0)$. This is far from shore in the sense that $k_x x_0 \ll 1$. The waves are "long" in the sense that $0 < k_x s x_0 \ll 1$ and they can therefore be treated as shallow water waves, which satisfy the dispersion relation $\omega^2 = c^2({k_x}^2 + {k_y}^2)$ with $c^2 = gH$.

   a) Find an integral expression, $y(x)$, for the path of the wavepacket.

   b) Without solving the exact integral, use approximation methods to determine the path of the wavepacket shortly after it passes through $(x_0, 0)$. Your answer for $y(x)$ will be a polynomial in x which you should give to $O(x^2)$ accuracy.

   c) Based on your answer in b) explain whether a wavepacket approaching a beach will veer toward or away from the beach.

2. **(/30)**
   Find the leading order behaviour (as $\lambda \to \infty$) of the following integrals:

   a) $\int_1^2 e^{-\lambda(t+\frac{1}{t})} \ln(1+t)\, dt$

   b) $\int_{-1}^1 e^{\imath\lambda t}(1-t^2)^{n-1}\, dt$.
   For what n is your answer valid?

   c) $\int_0^1 \sin[\lambda(t + \frac{1}{6}t^3 - \sinh t)] \cos t\, dt$

3. **(/30)**
   Find the full asymptotic series (as $\lambda \to \infty$) of the following integrals:

   a) $\int_0^{\pi/2} e^{-\lambda \sin^4 t} \sqrt{\sin t}\, dt$

   b) $\int_{-\infty}^{\infty} e^{-\lambda t^2} \ln(2+t^2)\, dt$

   c) $\int_0^1 e^{\imath\lambda t^3}\, dt$