# What Is Missing in Alberta Mathematics Education?

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# High School Mathematics Education

- The high-school preparation of students enrolled in first-year calculus courses at the University of Alberta has noticeably deteriorated.
- Associate Dean of Science, Prof. Gerda Devries, writes:

  *Students are not coming in with the same level of skills that they used to. Exams that were the norm 20 years ago are too difficult these days.*

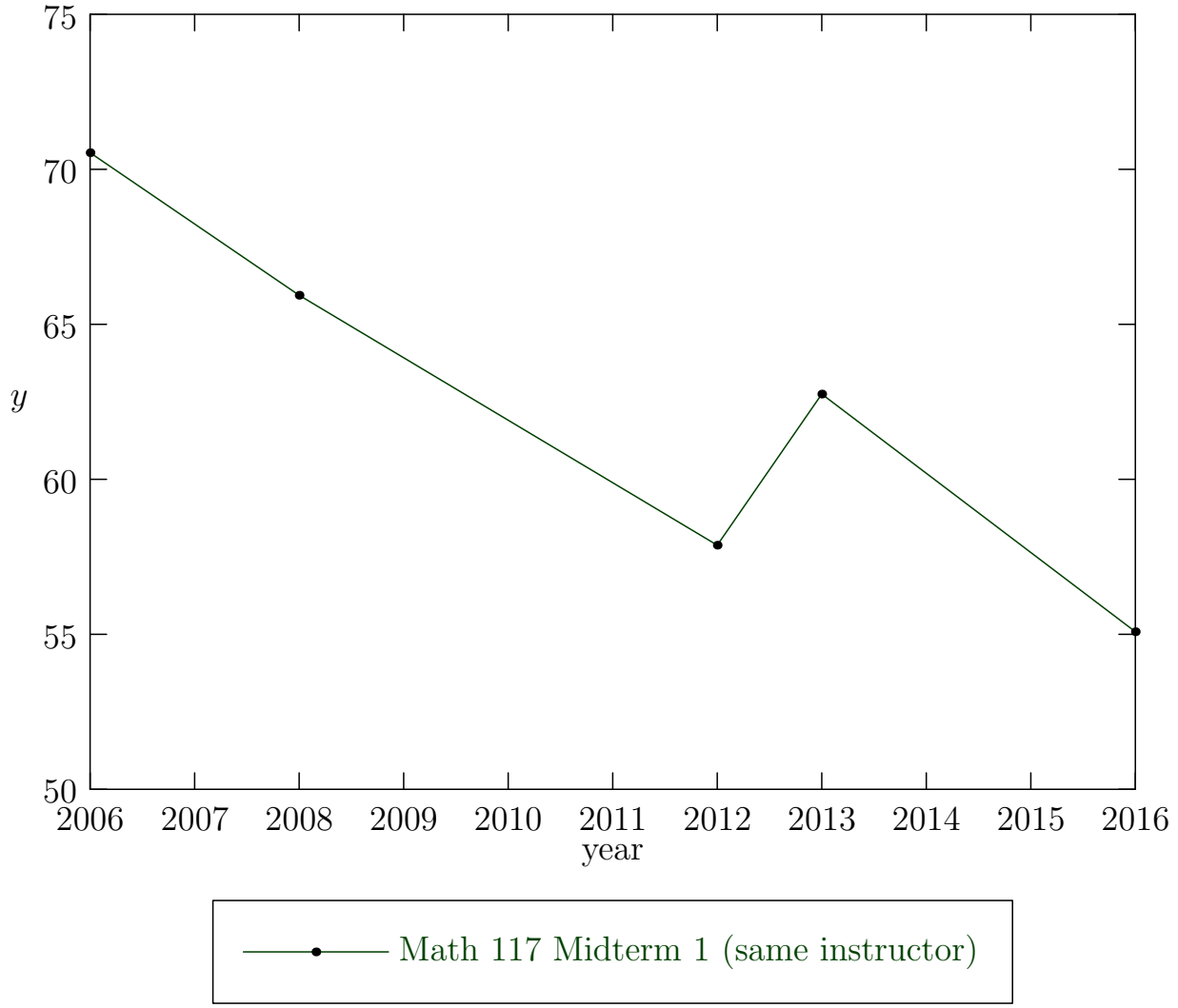
  *We are trying to carry more and more students who have very weak algebraic skills. It is not just the odd student who thinks that [all functions are linear].*

  *We have gone too far accommodating the growing cohort with weak foundational skills. We are starting to see issues in upper-level courses now, such as students taking a 3rd-year differential equations course not knowing how to graph f(t)=t.*

*Ten years ago, the big discussion [among Math 100 instructors] at the end of the term used to be where between 50 and 55 the cutoff should be for a passing grade. I believe that the discussion now is where between 45 and 50 that passing grade should be.*

- Former Associate Dean of Science, Prof. Brenda Leskiw, performed a study of individual Math 113/114 grades vs. high school mathematics grades. Surprisingly, she found no correlation!
- We need to revisit the 20–1 and 30–1 curriculum. These streams were supposed to be designed for post-secondary institutions, the majority stakeholder in these courses.
- A proposed 10–1 curriculum would provide earlier streaming.

# U of A Midterm Grades: 1st-Year Honours Calculus



- Mathematically talented Honours Math 117 students provide us with a reliable benchmark of what we are doing well and where we need to improve. The emphasis in these courses is on critical thinking, problem solving, and effective communication of their arguments.
- What is missing, and what can be removed, from the Alberta High School Mathematics curriculum?
- Let's ask these eminently qualified students...

# Missing Skills Reported by Honours Students

- *Elementary and middle school should focus on understanding of the basics.*
- *Long division of polynomials.*
- *Inequalities and absolute values.*
- *Completing the square.*
- *Circle geometry.*
- *Apart from solving simple systems of linear equations, where matrices are not even mentioned, linear algebra is completely absent from the high school curriculum. Vectors and matrices are of such fundamental importance in physics and engineering...*
- *Geometric series are done pretty well, as is trigonometry.*

# Missing Skills Reported by U of A Instructors

- Long division of polynomials: arbitrary degree. Don't introduce confusing restrictions to special cases like a trinomial divided by a binomial.
- Inequalities like $|x - a| < r$ means $-r < x - a < r$ means $a - r < x < a + r$.
- Completing the square.
- Logic:
  - and vs. or
  - for all vs. there exists
  - if vs. only if (cause vs. effect).
  - pigeon-hole principle: powerful for problem-solving.
- Euclidean geometry: a playground for teaching simple proofs.
- Parametric representations of lines and circles.

- Techniques for graphing simple functions.
- General addition of fractions should be covered by the end of elementary school.
- Textbooks should be refereed both by professionally trained mathematicians, who specialize in mathematics (the content), and math educators, who specialize in pedagogy (the delivery).

# What Should be Removed?

- Calculators in elementary school:

  – Priority should be teaching concepts not proficiency in technology.

  – Mathematics educators should spend the time to design tests that don't require calculators.

  – Students are often more proficient in technology than teachers since their generation has grown up with it.

- Estimation:

  – There is currently an over-emphasis on estimation. Estimation should be used as a check on calculations, not the other way around.

  – Estimation should be taught only after arithmetic proficiency has been achieved. Be aware that estimation is difficult to assess.

- Methods that only work in special cases: e.g. synthetic division, Sarrus's rule for $2 \times 2$ and $3 \times 3$ determinants.
- Measurement Experiments:

  – Proper measurement techniques belong in a good science course, not in mathematics.
- Dependency on Manipulatives:

  – Do manipulatives really belong in High School?

# What Can Be Scaled Back?

- Patterns and fabrics.
- Bar graphs are overemphasized:
  - Inherently one-dimensional.
  - Suitable for illustrating sequences but not for functions over the real line.
  - Causal relationships require two dimensions: an input (cause) and an output (effect).
- Visualization of data as it is currently taught in elementary and junior high school fits better into the science curriculum.

# Facts and Algorithms

- Effective problem solving: having basic facts and methods quickly retrievable by memory makes it easier to tackle big problems.
- Rote learning helps one acquire a good foundation of basic methods and facts that can be quickly retrieved for complex problem solving.
- Understanding algorithms is essential in my research (and throughout science and engineering).
- Algorithms, along with an understanding of the mathematics underlying them, are mandatory to prepare students for the 21st century.
- Engineers and scientists of tomorrow need to learn at an early age algorithms and number facts, the basic tools for their future profession.

# Recommendations of The Mathematics Curriculum Review Working Group

- Professional development:

  – Ideally, all teachers of junior high and high school mathematics should be mathematics majors or minors.

  – A designated mathematics specialist should be available in all K–6 schools to assist teachers who are generalists.

  – In Alberta, an elementary teacher is required to have three credits in mathematics...This differs from Quebec where elementary teachers are required to take a minimum of nine credits in mathematics, but most take between 12 and 15 credits.

- Numeracy should be considered an essential component in all subjects.

  – Emphasis should shift from the final answer to the process.

– Students must understand that mathematics takes practice. They should not be afraid to make mistakes.

– Students need to develop perseverance and confidence in their mathematics ability.

- Alberta Education [should] re-instate a written portion to the Mathematics 30–1 and Mathematics 30–2 diploma exams.
- The use of calculators for number operations should not be introduced until students have developed understanding of the concepts.
- Students should be able to sketch the graphs of linear, quadratic, and trigonometric functions by hand and interpret the graphs.
- Alberta Education should consider conducting part of the diploma exams without a calculator.
- Earlier teaching of the arithmetic of fractions without the use of a calculator.

- Acceleration of the treatment of linear relations (currently done in grades 7, 8, and 9) and placing greater emphasis on solving related problems by algebraic methods.
- Trigonometric, exponential, and logarithmic functions should be introduced sooner.
- Monitor readiness and success in post-secondary mathematics programs.

# Standard Algorithms [Robert Craigen]

- Reintroducing memorization of math facts and the algorithms are very good anchor points for improvement of the curriculum.
- The various algorithms are not learned all in one go—they tend to be acquired stepwise, in a scaffolded manner, over a period of years.
- Reinstate in elementary school:
  - least common denominator;
  - reducing fractions to simplest terms;
  - greatest common divisor;
  - lowest common multiple.

# Rote Learning

- Rote learning has not been eliminated with the curriculum changes since the Western Protocol, a precursor to the so-called 21st century learning model, was introduced in 2006.
- What happened is that rote learning has been downloaded on the parents, who resort to extra after-school instruction.
- The rise of Kumon and after-school learning is a very recent phenomenon.
- Not only do struggling students resort to after-school instruction, but a high percentage of top students receive extensive tutoring and rote learning after school.
- Working hours for even the youngest students often exceed 8 hours, compromising a healthy development. Student burnout and conflict are common side effects, just as in other cultures where students receive a high amount of after-school tutoring.

- An unintended side effect from moving away from rote learning during school hours is the creation of a two-class society:

  – The first class of parents either have the time to conduct extra hours of rote learning with their children or have the resources to pay for after-school tutoring.

  – The second class are families where both parents have to work and single parents who cannot afford tutoring. Many of the kids in these families are not doing well.

- A strong leadership in the government could change today's situation to the better, by learning from the results of the Western Protocol and having the courage to preserve some of the traditional, proven methods.

# Conclusions

- Students are not getting enough practice with basic arithmetic.
- The dependence on after-school tutoring should be reduced.
- The emphasis should be on quality, not quantity.
- Textbooks should be peer reviewed by mathematicians (Pearson textbooks contain many errors and confusing/nonstandard terminology).
- By emphasizing the importance of basic skills, mathematicians are advocating more, not less, problem solving.
- We all want students to understand what they are doing. It is not an either/or situation of memorization vs. understanding.
- We want to empower students with strong knowledge and skills, as lifelong learners who appreciate how math relates to the world around them.