What Is Missing in Alberta Mathematics Education?

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High School Mathematics Education

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• Associate Dean of Science, Prof. Gerda Devries, writes:

  Students are not coming in with the same level of skills that they used to. Exams that were the norm 20 years ago are too difficult these days.

  We are trying to carry more and more students who have very weak algebraic skills. It is not just the odd student who thinks that [all functions are linear].

  We have gone too far accommodating the growing cohort with weak foundational skills. We are starting to see issues in upper-level courses now, such as students taking a 3rd-year differential equations course not knowing how to graph \( f(t) = t \).
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• A proposed 10–1 curriculum would provide earlier streaming.
U of A Midterm Grades: 1st-Year Honours Calculus

Math 117 Midterm 1 (same instructor)
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What is missing, and what can be removed, from the Alberta High School Mathematics curriculum?

Let’s ask these eminently qualified students...
Missing Skills Reported by Honours Students

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• Completing the square.
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• Geometric series are done pretty well, as is trigonometry.
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• Parametric representations of lines and circles.
Techniques for graphing simple functions.
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• Textbooks should be refereed both by professionally trained mathematicians, who specialize in mathematics (the content), and math educators, who specialize in pedagogy (the delivery).
What Should be Removed?

• Calculators in elementary school:
  – Priority should be teaching concepts not proficiency in technology.
  – Mathematics educators should spend the time to design tests that don’t require calculators.
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• Estimation:
  – There is currently an over-emphasis on estimation. Estimation should be used as a check on calculations, not the other way around.
  – Estimation should be taught only after arithmetic proficiency has been achieved. Be aware that estimation is difficult to assess.
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• Measurement Experiments:
  
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• Dependency on Manipulatives:
  
  – Do manipulatives really belong in High School?
What Can Be Scaled Back?

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• Bar graphs are overemphasized:
  
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  – Suitable for illustrating sequences but not for functions over the real line.

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- Bar graphs are overemphasized:
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  - Causal relationships require two dimensions: an input (cause) and an output (effect).
- Visualization of data as it is currently taught in elementary and junior high school fits better into the science curriculum.
Facts and Algorithms

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• Engineers and scientists of tomorrow need to learn at an early age algorithms and number facts, the basic tools for their future profession.
Recommendations of The Mathematics Curriculum Review Working Group

• Professional development:

  – Ideally, all teachers of junior high and high school mathematics should be mathematics majors or minors.

  – A designated mathematics specialist should be available in all K–6 schools to assist teachers who are generalists.

  – In Alberta, an elementary teacher is required to have three credits in mathematics...This differs from Quebec where elementary teachers are required to take a minimum of nine credits in mathematics, but most take between 12 and 15 credits.
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• Numeracy should be considered an essential component in all subjects.
  
  – Emphasis should shift from the final answer to the process.
– Students must understand that mathematics takes practice. They should not be afraid to make mistakes.

– Students need to develop perseverance and confidence in their mathematics ability.

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- The use of calculators for number operations should not be introduced until students have developed understanding of the concepts.
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• Earlier teaching of the arithmetic of fractions without the use of a calculator.
• Acceleration of the treatment of linear relations (currently done in grades 7, 8, and 9) and placing greater emphasis on solving related problems by algebraic methods.
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• Monitor readiness and success in post-secondary mathematics programs.
• Reintroducing memorization of math facts and the algorithms are very good anchor points for improvement of the curriculum.
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• Reinstate in elementary school:
  – least common denominator;
  – reducing fractions to simplest terms;
  – greatest common divisor;
  – lowest common multiple.
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- Not only do struggling students resort to after-school instruction, but a high percentage of top students receive extensive tutoring and rote learning after school.

- Working hours for even the youngest students often exceed 8 hours, compromising a healthy development. Student burnout and conflict are common side effects, just as in other cultures where students receive a high amount of after-school tutoring.
An unintended side effect from moving away from rote learning during school hours is the creation of a two-class society:

- The first class of parents either have the time to conduct extra hours of rote learning with their children or have the resources to pay for after-school tutoring.

- The second class are families where both parents have to work and single parents who cannot afford tutoring. Many of the kids in these families are not doing well.
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• A strong leadership in the government could change today’s situation to the better, by learning from the results of the Western Protocol and having the courage to preserve some of the traditional, proven methods.
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• We want to empower students with strong knowledge and skills, as lifelong learners who appreciate how math relates to the world around them.