What Is Missing in High School Mathematics Education?

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High School Mathematics Education

- Over the last ten years, the high-school preparation of students enrolled in first-year Honours Calculus and Engineering Calculus courses at the University of Alberta has noticeably deteriorated.
- These mathematically talented students provide us with a reliable benchmark of what we are doing well and where we need to improve.
- Alberta Education is considering a proposal that would further degrade mathematics education in our province.
- A growing list of parents and educators are supporting a back-to-basics public backlash.
- What is missing, and what can be removed, from the Alberta High School Mathematics curriculum?
- Let's ask these eminently qualified students...

Feedback from the Front

- *Elementary and middle school should focus on understanding of the basics.*
- *I never actually learned how to long-divide polynomials even though everybody assumed I would know.*
- *A clearer understanding of the ideas behind inequalities and absolute values seem important and are rarely gone through in depth.*
- *Completing the square: My classes pretty well entirely passed over it and that's been hurting me.*
- *Circle geometry: properties of angles and chords were removed from the old curriculum. I think it would be good for Alberta Education to resurrect this topic, which surfaced sometimes in Math 117/118.*

- *Apart from solving simple systems of linear equations, where matrices are not even mentioned, linear algebra is completely absent from the high school curriculum. Vectors and matrices are of such fundamental importance in physics and engineering that it is crucial that students become acquainted with them early on. In my PHYS 144 class, the professor was baffled why we had not seen basic vector concepts, such as magnitude, addition, and unit vectors.*

- *Geometric series are done pretty well, as is trigonometry.*

- *It would be nice if high-school math bothered to prove things once in a while and demonstrate WHY anything works: after all, a lot of students will never use this stuff anyway, so it seems better to attain a more profound general understanding of how math works.*

So What Is Missing?

- Long division of polynomials: arbitrary degree.
- Inequalities like $|x - a| < r$ means $-r < x - a < r$ means $a - r < x < a + r$.
- Completing the square.
- Logic:
  - and *vs.* or
  - for all *vs.* there exists
  - if *vs.* only if (implications: cause *vs.* effect).
  - The pigeon-hole principle: powerful for problem-solving.
- Euclidean geometry: a playground for teaching simple proofs.
- Parametric representations of lines and circles.
- Techniques for graphing functions.

What Should be Removed?

- Calculators

  – Mathematics educators should spend the time to design tests that don't require calculators.

  – We want to test concepts, not hardware/software proficiency.

  – The younger generation should be teaching us technology, not the other way around!

- Measurement Experiments

  – Proper measurement techniques should be left to science courses.

- Manipulatives

  – Do these really belong in the High School Curriculum?

What Can Be Scaled Back?

- Patterns and fabrics.
- Bar graphs are overemphasized.
  - They are inherently one-dimensional.
  - They are suitable for illustrating sequences but not for functions over the real line.
  - Causal relationships require two dimensions: an input (cause) and an output (effect).

Facts and Algorithms

- Learning math is analogous to learning a new language: a large vocabulary is essential.
- Rote learning is a proven way to acquire basic vocabulary.
- Sure, one could instead look words up in a dictionary. But one will never become fluent this way.
- Effective problem solving: having basic facts and methods quickly retrievable by memory makes it easier to tackle big problems.
- Rote learning helps one acquire a good foundation of basic methods and facts that can be quickly retrieved for complex problem solving.
- Engineers and scientists of tomorrow need to learn at an early age algorithms and number facts, the basic tools for their future profession.

Strategies

- Strategies and tricks are interesting for the brightest students.
- However, the average student (including future professionals) will be better served by learning the time-tested algorithms for arithmetic computation that we learned as children and continue to use in our daily lives.

Rote learning

- Rote learning has not been eliminated with the curriculum changes since the Western Protocol, a precursor to the so-called 21st century learning model, was introduced in 2006.
- What happened is that rote learning has been downloaded on the parents, who resort to extra after-school instruction: Kumon and other after-school tutoring. The rise of Kumon and after-school learning is a very recent phenomenon.
- As a parent, I was surprised to learn that in present classrooms not only the struggling students resort to after-school instruction, but a high percentage of top students receive extensive tutoring and rote learning after school.
- Working hours for even the youngest students often exceed 8 hours, compromising a healthy development. Student burnout and conflict are common side effects, just as in other cultures where students receive a high amount of after-school tutoring.

- An unintended side effect from moving away from rote learning during school hours is the creation of a two-class society:

  – The first class of parents either have the time to conduct extra hours of rote learning with their children or have the resources to pay for after-school tutoring.

  – The second class are families where both parents have to work and single parents who cannot afford tutoring. Many of the kids in these families are not doing well.

- A strong leadership in the government could change today's situation to the better, by learning from the results of the Western Protocol and having the courage to preserve some of the traditional proven methods.

Conclusions

- Students are not getting enough practice with basic arithmetic.
- The emphasis should be on quality, not quantity.
- By emphasizing the importance of basic skills, mathematicians are advocating more, not less, problem solving.
- We all want students to understand what they are doing. It is not an either/or situation of memorization *vs.* understanding.
- We do not want to innovate just for the sake of innovation.