1. Let \( \{S_n\}_{n=1}^{\infty} \) be a sequence of sets. Define
\[
\liminf_{n \to \infty} S_n = \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} S_n
\]
and
\[
\limsup_{n \to \infty} S_n = \bigcap_{k=1}^{\infty} \bigcup_{n=k}^{\infty} S_n.
\]
Show that
\[
\liminf_{n \to \infty} S_n \subset \limsup_{n \to \infty} S_n.
\]

2. Let \( \{S_j\}_{j=1}^{\infty} \) be a sequence of sets. Define \( T_1 = S_1 \) and
\[
T_j = S_j \setminus \bigcup_{i=1}^{j-1} S_i, \quad j = 2, 3, \ldots
\]
Prove that \( \{T_j\}_{j=1}^{\infty} \) is a sequence of pairwise disjoint sets satisfying
\[
\bigcup_{j=1}^{n} T_j = \bigcup_{j=1}^{n} S_j, \quad \forall n \in \mathbb{N}.
\]

3. Suppose that \( A \) and \( B \) are subsets of \( \mathbb{R}^d \), with \( m^*(A) < \infty \) and \( m^*(B) < \infty \). Prove that
\[
|m^*(A) - m^*(B)| \leq m^*(A \Delta B).
\]

4. Give examples of
   (a) a bounded countable union of Jordan measureable sets that is not Jordan measureable;
   (b) a bounded countable intersection of Jordan measureable sets that is not Jordan measureable.
5. Determine whether each of the following sets is countable. Justify your answers.

(a) The set of all mappings from $1, \ldots, N$ to $\mathbb{N}$, where $N$ is a fixed positive integer.

(b) The set of all mappings from $\mathbb{N}$ to $[0, 1]$.

(c) The set of all mappings $f$ from $\mathbb{N}$ to $[0, 1]$ that are “eventually zero”; i.e. there is a positive integer $N$ such that $f(n) = 0$ for all $n > N$.

(d) The set of all finite subsets of $\mathbb{N}$.

6. If $\{x_\alpha\}_{\alpha \in A}$ is a collection of numbers $x_\alpha \in [0, \infty]$ such that $\sum_{\alpha \in A} x_\alpha < \infty$, show that $x_\alpha = 0$ for all but at most countably many $\alpha \in A$, even if $A$ is itself uncountable.

7. Let $S$ be a set. Prove that there is no surjective map from $f : S \mapsto \mathcal{P}(S)$. Hint: Consider the set $T \doteq \{s \in S : s \notin f(s)\}$.

8. Let $S$ be a countable set. Show that

$$\mathfrak{F}(S) \equiv \{F \subset S : F \text{ is finite}\}$$

is a countable subset of $\mathcal{P}(S)$.