1. Let \( \{S_n\}_{n=1}^{\infty} \) be a sequence of sets. Define
\[
\liminf_{n \to \infty} S_n = \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} S_n
\]
and
\[
\limsup_{n \to \infty} S_n = \bigcap_{k=1}^{\infty} \bigcup_{n=k}^{\infty} S_n.
\]
Show that
\[
\liminf_{n \to \infty} S_n \subset \limsup_{n \to \infty} S_n.
\]

2. Let \( \{S_j\}_{j=1}^{\infty} \) be a sequence of sets. Define \( T_1 = S_1 \) and
\[
T_j = S_j \setminus \bigcup_{i=1}^{j-1} S_i, \quad j = 2, 3, \ldots
\]
Prove that \( \{T_j\}_{j=1}^{\infty} \) is a sequence of pairwise disjoint sets satisfying
\[
\bigcup_{j=1}^{n} T_j = \bigcup_{j=1}^{n} S_j, \quad \forall n \in \mathbb{N}.
\]

3. Determine whether each of the following sets is countable. Justify your answers.
   (a) The set of all mappings from \( \{1, \ldots, N\} \) to \( \mathbb{N} \), where \( N \) is a fixed positive integer.
   (b) The set of all mappings from \( \mathbb{N} \) to \( \{0, 1\} \).
   (c) The set of all mappings \( f \) from \( \mathbb{N} \) to \( \{0, 1\} \) that are “eventually zero”; i.e. there is a positive integer \( N \) such that \( f(n) = 0 \) for all \( n > N \).
   (d) The set of all finite subsets of \( \mathbb{N} \).

4. If \( \{x_\alpha\}_{\alpha \in A} \) is a collection of numbers \( x_\alpha \in [0, \infty] \) such that \( \sum_{\alpha \in A} x_\alpha < \infty \), show that \( x_\alpha = 0 \) for all but at most countably many \( \alpha \in A \), even if \( A \) is itself uncountable.
5. Is the countably infinite Cartesian product of the two-point set \{0, 1\} countable? Justify your answer.

6. If \(E, F \subset \mathbb{R}^d\) are elementary sets, show that the union \(E \cup F\), the intersection \(E \cap F\), the set theoretic difference \(E \setminus F\) \(= \{x \in E : x \notin F\}\), and the symmetric difference \(E \triangle F = (E \setminus F) \cup (F \setminus E)\) are also elementary. If \(x \in \mathbb{R}^d\), show that the translation \(E + x\) \(= \{y + x : y \in E\}\) is also an elementary set.

7. Give examples of
   (a) a bounded countable union of Jordan measureable sets that is not Jordan measureable;
   (b) a bounded countable intersection of Jordan measureable sets that is not Jordan measureable.

8. Suppose that \(A\) and \(B\) are subsets of \(\mathbb{R}^d\), with \(m^*(A) < \infty\) and \(m^*(B) < \infty\). Prove that
   \[|m^*(A) - m^*(B)| \leq m^*(A \Delta B)\].