Math 373: Mathematical Programming and Optimization I
Fall, 2023 Assignment 3
November 8, due November 25

1. Let \( P = \{ x \in \mathbb{R}^n : a_i^T x \geq b_i, i = 1, \ldots, m \} \) be a convex polyhedron.
   Suppose that \( u \) and \( v \) are distinct basic feasible solutions that satisfy \( a_i^T u = a_i^T v = b_i \) for \( i = 1, \ldots, n - 1 \), so that \( u \) and \( v \) are adjacent, where \( a_1, \ldots, a_{n-1} \) are linearly independent.
   Let \( L_1 = \{ t u + (1 - t) v : 0 \leq t \leq 1 \} \) be the segment that joins \( u \) and \( v \) and \( L_2 = \{ w \in P : a_i^T w = b_i, i = 1, \ldots, n - 1 \} \). Prove that \( L_1 = L_2 \).

2. If a linear programming problem in standard form has a non-degenerate basic feasible solution that is optimal, prove that the dual problem has a unique optimal solution. Hint: consider complementary slackness.