GAME THEORY

Rock | Paper | Scissors
Player 1 & Player 2

Rock > Scissors > Paper > Rock

Payoff Matrix for player 2 column

\[
A = \begin{bmatrix}
0 & -1 & 1 \\
1 & 0 & -1 \\
1 & 1 & 0
\end{bmatrix}
\]

Player 1 row

\[
B = \begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}
\]

pure strategy

\[R \quad P \quad S\]

Strategy vector 1: \[x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 1, x_1, x_2, x_3 \geq 0\]

Mixed strategy

\[x = (x_1, x_2, x_3) \quad \text{subject to} \quad x_1 + x_2 + x_3 = 1, x_1, x_2, x_3 \geq 0\]
Player 1: R S R S R P R R R R 
Player 2: R S R P S R R S R R P R 

\[ \begin{pmatrix} \frac{3}{5}, 1, & 1/5, & 5 \end{pmatrix} \quad \text{1: } R=6, P=2, S=2 \\
\begin{pmatrix} 1/2, & 1/5, & 3/10 \end{pmatrix} \quad \text{2: } R=5, P=2, S=3 \\
\text{mixed strategies} \]

Prisoner's Dilemma's 
2 thieves A & B 
Arrested 
A & B confess: 2 year each 
only A confesses: 3 years for B 
only B confesses: 3 years for A 
cooperating: neither confess: 1 year each 

Individual rationality does not lead to collective rationality.
Tragedy of the Commons:
- overgrazing on common land
- overfishing

Battle of the Sexes

\[
\begin{bmatrix}
3 & 0 \\
-1 & 1
\end{bmatrix}
\quad \text{Man} = \quad \begin{bmatrix}
1 \\
0 & 3
\end{bmatrix}
\quad \text{Woman}
\]

Best payoff is when both agents cooperate.

Co-ordinating Game

Snowdrift problem

Anti-coordinate: best to pick opposite strategy of other player

Payoff

If other player shovels, share the job; if not, shovel on your own; neither shovels.
\( C = \text{cooperate} \)
\( D = \text{defect (don't cooperate with partner)} \)

\[
\begin{align*}
C &:\quad \begin{cases} 
   C & r = 3 \\
   r - c & r > 0 \\
   r - c + 1 & c = 2 \\
   0 & c < r
\end{cases} \\
D &:\quad r = 3
\end{align*}
\]
Classification of 2x2 games

Payoff matrix \( A = \begin{bmatrix} R & S \\ T & P \end{bmatrix} \)

1) Prisoner's Dilemma:
   \( T > R > S > P \)

2) Coordination Game:
   \( R > T \quad P > S \)

3) Snowdrift Game:
   \( T > R \quad S > P \)