1. Show that
\[ \gamma: [0, 1] \rightarrow \mathbb{R}^2, \quad t \mapsto \begin{cases} (1, t \cos \left( \frac{\pi}{t} \right)) & \text{if } t \in (0, 1], \\ (1, 0) & \text{if } t = 0 \end{cases} \]
defines a curve that fails to be rectifiable. (*Hint: Consider partitions \( 0 < \frac{1}{2^m} < \frac{1}{2^{m-1}} < \cdots < \frac{1}{3} < \frac{1}{2} < 1 \).)

2. Let \( \Phi \) be a surface in \( \mathbb{R}^3 \) with parameter domain \( K \subset \mathbb{R}^2 \), let \( \gamma: [a, b] \rightarrow K \) be a \( C^1 \) curve, and let \( \alpha := \Phi \circ \gamma \). Show that \( \alpha'(t) \) is orthogonal to \( N(\gamma(t)) \) for each \( t \in [a, b] \). Interpret this result geometrically.

3. Let \( \Phi \) and \( \Psi \) be \( C^2 \)-surfaces with parameter domain \( K \), which is a normal region, such that \( \Phi|_{\partial K} = \Psi|_{\partial K} \), and let \( f: V \rightarrow \mathbb{R}^3 \) be continuously differentiable where \( V \subset \mathbb{R}^3 \) is open and contains \( \{\Phi\} \cup \{\Psi\} \). Show that
\[ \int_\Phi \text{curl} \cdot \mathbf{n} \, d\sigma = \int_\Psi \text{curl} \cdot \mathbf{n} \, d\sigma. \]

4. Let \( V \) be a normal domain with boundary \( S \) such that \( N \neq 0 \) on \( S \) and let \( f \) and \( g \) be \( \mathbb{R} \)-valued \( C^2 \) functions on an open set containing \( V \).
   (a) Prove Green’s First Formula:
   \[ \int_V (\nabla f) \cdot (\nabla g) + \int_V f \Delta g = \int_S f \mathbf{D}_n g \, d\sigma. \]
   *Hint: Apply Gauß’ Theorem to the vector field \( f \nabla g \).*
   (b) Prove Green’s Second Formula:
   \[ \int_V (f \Delta g - g \Delta f) = \int_S (f \mathbf{D}_n g - g \mathbf{D}_n f) \, d\sigma. \]

5. Let \( \emptyset \neq U \subset \mathbb{R}^3 \) be open, and suppose that \( f \in C^2(U, \mathbb{R}) \) is harmonic, i.e., satisfies \( \Delta f = 0 \). Let \( V \subset U \), \( S \), and \( n \) be as in Question 4. Show that
   (a) \[ \int_S D_n f \, d\sigma = 0. \]
(b) \[ \int_S f D_n f \, d\sigma = \int_V |\nabla f|^2. \]