

Comparison of numerical approximation methods and their errors

Consider our example integral:

$$\int_0^1 \frac{4}{1+x^2} dx$$

Calculated values using midpoint, trapezoid and Simpson's rules:

n	M_n	T_n	S_n
4	3.146800518	3.131176471	3.141592502
8	3.142894730	3.138988494	3.141592651
16	3.141918174	3.140941612	3.141592654
32	3.141674034	3.14129893	3.141592654

Error of midpoint, trapezoid and Simpson's rules:

n	M_n	T_n	S_n
4	0.005207864	0.010416183	0.000000152
8	0.001302076	0.002604160	0.000000003
16	0.000325520	0.000651042	0.0
32	0.000081380	0.000162761	0.0

Note: You can calculate these values using Maple:

```
with(Student[Calculus1]):  
#to calculate the approximation  
ApproximateInt(4/(1+x^2), x = 0 .. 1, method = midpoint, partition = 4);  
#to calculate the error  
abs(evalf(Pi-%));
```