

## Logarithmic derivatives, rates of change and related rates

**Material covered:** Chapter 3, Sections 3.6, 3.7, 3.9

Derivatives of logarithmic functions

$$\begin{aligned}\frac{d}{dx}(\log_a x) &= \frac{1}{x \ln a} \\ \frac{d}{dx}(\ln x) &= \frac{1}{x} \\ \frac{d}{dx}(\ln g(x)) &= \frac{g'(x)}{g(x)} \\ \frac{d}{dx}(\ln |x|) &= \frac{1}{x}\end{aligned}$$

### Steps in logarithmic differentiation

1. Given  $y = f(x)$ , take natural log of both sides and simplify using log rules
2. Implicitly differentiate both sides with respect to  $x$
3. Solve the resulting equation for  $\frac{dy}{dx}$

**The number  $e$  as a limit**

$$e = \lim_{x \rightarrow 0} (1 + x)^{1/x} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

## Rates of change

**Recall:** The average rate of change of  $y$  with respect to  $x$  can be written as the difference quotient

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

while the instantaneous rate of change of  $y$  with respect to  $x$  can be written using Leibniz notation as

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

## Related rates

### **Steps for solving related rate problems**

1. Read the problem carefully and make a diagram if possible.
2. Make a table with two columns: one for given quantities and the other for unknown quantities. Don't forget to include units.
3. Assign symbols to all quantities that vary with time and express given information and unknowns in terms of derivatives.
4. Write an equation that associates the variables with one another. Try to eliminate variables using geometry.
5. Use rules of differentiation to differentiate both sides of the equation with respect to  $t$ .
6. Substitute in the given information to solve for the unknown rate.