What is the right strategy of integration?

What is the strategy and why it is useful?

- Strategy — a **general** plan of actions that leads to the goal.
- To work out strategy we can drop out constant factors, usually they are inessential for the strategy.
- Look at the types of integrals. What transformations can be done to the given integral and to what other integrals it can be reduced? Are they simpler? Or closer to a tabular integral?
- After determining the right strategy, we can do all the details with all factors and dropped terms

What we can do?

- Simplify
- Look for good substitittutions
- Classify, recognize typical patterns. To do it we need to know basic integrals and derivatives and basic techniques.
- Never give up, try all possible "branches of strategy" if several actions look promising.

Examples

$$\int \frac{1}{\sin^2 x + \cos^2 x} dx = \int dx$$

$$\int \frac{\sin x}{1+\cos^2 x} dx \to [u = \cos x] \quad \int \frac{du}{1+u^2}$$

$$\int x^3 \ln x \, dx \to \text{(IBP)} \quad \ldots + \int x^4 \frac{1}{x} dx$$

$$\int \frac{x}{\sqrt{3-x^4}} dx \to \left[u = x^2\right] \quad \int \frac{du}{\sqrt{3-u^2}} \to \left[u^2 = 3z^2\right] \quad \int \frac{dz}{\sqrt{1-z^2}}$$

$$\int e^{x+e^x} \, dx = \int e^{e^x} \, e^x dx \to [u = e^x] \quad \int e^u du$$

$$\int e^{x^{1/3}} dx \to \left[u = x^{1/3}\right] \quad \int e^u u^2 du \to \text{(IBP)} \quad \ldots + \int u e^u du \to \ldots + \int e^u du$$

$$\int \ln\left(1+x^2\right)\,dx \to \text{(IBP)} \quad \ldots + \int \frac{2x}{1+x^2}x dx \to \int dx + \int \frac{1}{1+x^2}dx$$

$$\int t^3 e^{-2t}dt \to \text{(IBP)} \quad \ldots + \int t^2 e^{-2t}du \to \ldots + \int te^{-2t}du \to \ldots + \int e^{-2t}du$$

$$\int \sqrt{z}\left(z+z^{1/3}\right)\,dz \to \int z^{3/2}dz + \int z^{5/6}dz$$

$$\int \cot x\, \ln\left(\sin x\right) dx \to [u=\sin x] \quad \int \frac{1}{u}\ln u\, du \to [v=\ln u] \quad \int v dv$$

$$\int x^2 \tan^{-1} x\, dx \to \text{(IBP)} \quad \ldots + \int x^3 \frac{1}{1+x^2}dx = \int x^2 \frac{1}{1+x^2}x dx \to$$

$$\to \left[u = x^2+1\right] \quad \int \frac{u-1}{u}du$$

$$\int \frac{\tan^{-1}\left(\sqrt{t}\right)}{\sqrt{t}}dt = \left[x=\sqrt{t},\, t=x^2\right] \quad \int \frac{\tan^{-1} x}{x}2x dx = 2\int \tan^{-1} x\, dx \to$$

$$\to \text{(IBP)} \quad \ldots + \int \frac{1}{1+x^2}x dx \to \left[z = 1+x^2\right] \int \frac{1}{z}dz$$

$$\int \frac{x^4}{x^{10}+16}dx = \left[u = x^5\right] \int \frac{du}{u^2+16} \to \left[u^2 = 16y^2\right] \quad \int \frac{dy}{1+y^2}$$

$$\int_{-1}^{1} x^8 \sin x\, dx = 0$$

$$\int \frac{x^3}{(x+1)^{10}}\,dx = [u = x+1] \quad \int \frac{(u-1)^3}{u^{10}}du$$