Asymptotic Geometric Analysis

July 1 – July 6, 2019

SCHEDULE

Monday, July 1

9:00 – 9:25 Registration
9:25 – 9:30 Opening
9:30 – 10:20 **V. Milman**, *"Flowers and Non-Linear Constructions in Convex Geometry."*
10:30 – 10:55 **B. Gonzalez Merino**, *"On Hermite-Hadamard and Jensen inequalities."*
11:00 – 11:30 Coffee Break
11:30 – 11:55 **M. Naszodi**, *"Extensions of results on approximate John decomposition."*
12:00 – 12:25 **M. Fradelizi**, *"Inequalities on mixed volumes and analogies with information theory."*

12:30 – 14:30 Lunch

14:30 – 15:20 **A. Giannopoulos**, *"Norms of weighted sums of log-concave random vectors."*
15:30 – 15:55 **C. Saraglou**, *"Constant parts of a function via isotropicity of its sections."*
16:00 – 16:30 Coffee Break
16:30 – 16:55 **A. Kolesnikov,** *"On transportation inequalities on the sphere and log Brunn-Minkowski problem."*
17:00 – 17:25 **N. Zhang**, *"Topics on Milman's problem."*
17:30 – 17:55 **T. Hack**, *"Randomized Urysohn-type inequalities."*

18:15 **RECEPTION** *at the Euler Institute*

Tuesday, July 2

9:30 – 10:20 **B. Kashin**, *"Some theorems on the restriction of operator to coordinate subspace."*
10:30 – 10:55 **T. Tkocz**, *"On a certain convexity property for sections of the cross-polytope."*
11:00 – 11:30 Coffee Break
11:30 – 11:55 **D. Zaporozhets**, *"Generalized Busemann inequality."*
12:00 – 12:25 **K. Tatarko**, *"On the reverse isoperimetric problem."*

12:30 – 14:30 Lunch

14:30 – 14:55 **C. Schütt**, *"Half space depth and floating body."*
15:00 – 15:25 **H. Rauhut**, *"On the geometry of random polytopes generated by heavy-tailed random vectors."*
15:30 – 16:00 Coffee Break
16:00 – 16:25 **P. Valettas**, *"Concentration and Convexity."*
16:30 – 16:55 **P. Nayar**, *"Hadamard products and concentration of log-concave measures."*
17:00 – 17:25 **C.H. Jiménez**, *"A functional version of the Busemann-Petty centroid inequality and its connections with Sobolev inequalities."*

Wednesday, July 3

9:30 – 10:20 **K. Tikhomirov**, *"Singularity of random Bernoulli matrices."*
10:30 – 10:55 **M. Strzelecka**, *"Estimates of norms of log-concave random matrices with dependent entries."*
11:00 – 11:30 Coffee Break
11:30 – 11:55 **O. Friedland**, *"Playing billiards with gaussian functions."*
12:00 – 12:25 **P. Kniefacz**, *"Sharp Sobolev inequalities via projection averages."*

12:30 – 14:30 Lunch

14:30 – 14:55 **S. Myroshnychenko**, *"On recognizing convex bodies from their shadows."*
15:00 – 15:25 **S. Sadovsky**, *"Mixed volume inequalities on a special class of convex bodies."*
15:30 – 15:55 **O. Reinov**, *"On products of s-nuclear operators, $s \in (0, 1]$."*
16:00 – 16:30 Coffee Break
16:30 – 16:55 **E. Abakumov**, *"On Chui's Conjecture."*
17:00 – 17:25 **E. Safronenko**, *"New extremal convex bodies for affine measures of symmetry."*
17:30 – 17:55 **I. Vasilyev**, *"On the minimizing rectifiable G chains of dimension 2 and codimension 1 in finite dimensional Banach spaces."*

19:30 **BANQUET** at "Marsala," Bolshoy prospekt P.S. 84 (near metro station "Petrogradskaya"), http://marsalaspb.ru

Thursday, July 4

10:00 **EXCURSION** to Peterhof Lower Park and Kronshtadt.
Bus will be waiting at 10:00 in front of Anderson hotel.
Excursion will take approximately 8-9 hours.

Friday, July 5

9:30 – 10:20 **A. Eskenazis**, *"Progress on Enflo's conjecture."*
10:30 – 10:55 **A. Gusakova**, *"Limiting theorems for Poisson-Delaunay tessellation."*
11:00 – 11:30 Coffee Break
11:30 – 12:00 **G. Bianchi**, *"Convergence of symmetrization processes."*
12:00 – 12:25 **S. Dann**, *"Extensions of dual affine quermassintegrals to flag manifolds."*

12:30 – 14:30 Lunch

14:30 – 14:55 **S. Reisner**, *"Ellipsoids are the only local maximizers of the volume product."*
15:00 – 15:25 **D. Alonso-Gutiérrez**, *"Zhang's inequality for log-concave functions."*
15:30 – 16:00 Coffee Break
16:00 – 16:25 **F. Besau**, *"Polytopal Approximation in the dual Brunn–Minkowski Theory."*
16:30 – 16:55 **O. Rusakov**, *"Limits of special random walks in polygones and a class of stochastic fractals."*

Saturday, July 6

9:30 – 10:20 **M. Rudelson**, *"Circular law for sparse random matrices."*
10:30 – 10:55 **D. Ryabogin**, *"On a local version of the fifth Busemann-Petty problem."*
11:00 – 11:30 Coffee Break
11:30 – 11:55 **G. Hofstätter**, *"Blaschke-Santaló inequalities for Minkowski Endomorphisms."*
12:00 – 12:25 **A. Zvavitch**, *"Volume product and Lipschitz-free Banach spaces."*

ABSTRACTS

Evgeny Abakumov

University Paris Est, Marne-la-Vallée, France

Title: *On Chui's Conjecture.*

Abstract: In 1971, C.K.Chui conjectured that the average fields strength in the unit disc due to unit point masses on the unit circle is minimal for the uniform distribution of masses. We discuss the analogous lower bound problem for weighted L^2 norms.

This is a joint work with A. Borichev and K. Fedorovskiy.

David Alonso-Gutiérrez

University of Zaragoza, Zaragoza, Spain

Title: *Zhang's inequality for log-concave functions.*

Abstract: Zhang's inequality states that among all the n-dimensional convex bodies the affine invariant quantity $|K|^{n-1}|\Pi^* K|$, where $\Pi^* K$ denotes the polar projection body of K, is minimized by the simplex. This inequality reverts Petty projection inequality, which states that this quantity is maximized by the Euclidean ball. We will extend Zhang's inequality to the context of log-concave function, proving that for any integrable log-concave function

$$\int_{\mathbb{R}^{2n}} \min\{f(x), f(y)\} dydx \leq 2^n n! \|f\|_1^{n+1} |\Pi^*(f)|,$$

where $\Pi^*(f)$ denotes the polar projection body of the log-concave function f. Whenever $f(x) = e^{-\|x\|_K}$ with K a convex body containing the origin Zhang's inequality is recovered.

Florian Besau

University of Frankfurt, Frankfurt, Germany

Title: *Polytopal Approximation in the dual Brunn–Minkowski Theory.*

Abstract: The question of approximating the volume of a convex body by an inscribed polytope with a bounded number of vertices has been raised frequently and is by now very classical. In general, this question is considered to be very hard, for example, for the unit ball in three dimensions the best approximating inscribed polytopes are explicitly known only if the number of vertices is very small. Hence, asymptotic estimates for when the number of vertices goes to infinity have been of considerable interest. Remarkably, Gruber obtained a limit theorem in arbitrary dimension for the volume difference between a fixed smooth convex body and the best approximating polytope as the number of vertices goes to infinity.

A probabilistic approach to the question of approximating the volume is to draw a fixed number of points independently and at random from the inside of the convex body and construct the convex hull. This gives a random polytope that will converge to the original convex body almost surly when the number of points goes to infinity. In particular, this construction is useful if it is very hard to obtain information about the boundary of the convex body, but if it is comparable easy to check whether a point is inside of the convex body or not.

If we instead assume complete information about the boundary of our convex body, then one may a prior choose random points already from the boundary to obtain better approximating polytopes. This setup was considered by Schütt & Werner and a limit theorem for the expected volume of these random polytopes was established. Remarkably, when choosing the optimal density on the boundary of the convex body, i.e., a density such that the limit of the expected volume difference is minimal, then the expected volume of the random polytopes behaves asymptotically similar to the volume of the best approximating polytope.

In this talk I will present our results for the asymptotic best and random approximation of convex bodies by polytopes with a bounded number of vertices or facets, where instead of the volume we are interested in the dual volumes (or dual quermassintegrals) which originate from Lutwak's dual Brunn–Minkowski theory. This talk is based on joint work together with Steven Hoehner and Gil Kur; arXiv:1905.08862.

Gabriele Bianchi

University of Florence, Florence, Italy

Title: *Convergence of symmetrization processes.*

Abstract: It is known that there are sequences (H_m) of subspaces such that if K is a convex body and $\diamondsuit_H$ denotes Steiner or Minkowski symmetrization with respect to H then the sequence $\left(\diamondsuit_{H_m}\diamondsuit_{H_{m-1}}\cdots\diamondsuit_{H_1}K\right)$ converges to a ball.

- We study the same phenomenon for different symmetrizations $\diamondsuit_H$ and we also study the dependence on the symmetrization of the "rounding" sequence (H_m) of subspaces. We find conditions on a generic symmetrization $\diamondsuit_H$ that grant that any sequence of subspaces (H_m) which is "rounding" for Steiner or Minkowski symmetrizations is also "rounding" for $\diamondsuit_H$.
- Is it more difficult to "round" a compact set than a convex set? We prove that, at least when dealing with Steiner, Schwartz or Minkowski symmetrizations, a sequence (H_m) of subspaces is "rounding" in the class of compact sets if and only if it is so in the class of convex bodies.
- A problem that is relevant in this research and on which we presents some results is the following: *Let $1 \leq i \leq n-1$, let $F_1, \dots, F_m$ be i-dimensional subspaces of $\mathbb{R}^n$ and let $K \subset \mathbb{R}^n$ be a convex body. For which choices of $F_1, \dots, F_m$ does the* reflection symmetry *of K w.r.t. each F_j forces K to be a ball? And what about* rotational simmetry *w.r.t. each F_j?*

This is a joint research with R. J. Gardner and P. Gronchi.

Susanna Dann

Universidad de los Andes Carrera, Bogota, Colombia

Title: *Extensions of dual affine quermassintegrals to flag manifolds.*

Abstract: In this talk we discuss generalizations of dual affine quermassintegrals as averages on flag manifolds (where the Grassmannian can be considered as a special case) and extend all known results to this new setting.

This is a joint work with Grigoris Paouris and Peter Pivovarov.

Alexandros Eskenazis

Princeton University, Princeton, USA

Title: *Progress on Enflo's conjecture.*

Abstract: In modern terminology, Enflo's conjecture (1978) asserts that a Banach space X has Rademacher type p if and only if X satisfies a metric property called Enflo type p. Loosely speaking, the conjecture suggests that all X-valued functions on the Hamming cube satisfy a dimension independent L_p Poincaré inequality if and only if the same inequality holds merely for linear functions. In his 1986 work, Pisier showed that Banach spaces of Rademacher type p have Enflo type q for every $q < p$ and proved the endpoint Enflo type p inequality with an additional logarithmic factor in the dimension of the Hamming cube. In this talk, I shall present joint work in progress with A. Naor, in which we improve Pisier's bound for Banach spaces which admit an equivalent uniformly convex norm. The proof relies on (either new or recently proven) vector valued Littlewood-Paley-Stein theory on the Hamming cube.

Matthieu Fradelizi

University Paris Est, Marne-la-Vallée, France

Title: *Inequalities on mixed volumes and analogies with information theory.*

Abstract: We present analogues in convex geometry of entropy power-type inequalities in particular a local version of the Alexandrov-Fenchel inequality and its connection with the comparison of the volume of a convex set and the volume of its projections.

This is a joint work with Mokshay Madiman and Artem Zvavitch.

Omer Friedland

University Pierre and Marie Curie, Paris, France

Title: *Playing billiards with gaussian functions.*

Abstract: Rational polygonal billiards are one of the key models among the larger class of pseudo-integrable billiards. Their billiard flow may be lifted to the geodesic flow on a translation surface. Whereas such classical billiards have been much studied in the literature, the analogous quantum billiards have received much less attention. This talk is concerned with a conjecture of Bogomolny and Schmit who proposed in 2004 that the eigenfunctions of the Laplacian on rational polygonal billiards ought to become localized along a finite number of vectors in momentum space, as the eigenvalue tends to infinity. For any given momentum vector $\xi_0 \in \mathbb{S}^1$ we construct a continuous family of quasimodes which gives rise to a semi-classical measure whose projection on momentum space is supported on the orbit $D\xi_0$, where D denotes the dihedral group associated with the rational polygon.

Apostolos Giannopoulos

National and Kapodistrian University of Athens, Athens, Greece

Title: *Norms of weighted sums of log-concave random vectors.*

Abstract: Let K be a centrally symmetric convex body in $\mathbb{R}^n$. For any s-tuple $\mathcal{C} = (C_1, \ldots, C_s)$ of centrally symmetric convex bodies C_j in $\mathbb{R}^n$ we consider the norm on $\mathbb{R}^s$, defined by

$$\|\mathbf{t}\|_{\mathcal{C},K} = \frac{1}{\prod_{j=1}^s \mathrm{vol}_n(C_j)} \int_{C_1} \cdots \int_{C_s} \Big\| \sum_{j=1}^s t_j x_j \Big\|_K dx_1 \cdots dx_s,$$

where $\mathbf{t} = (t_1, \dots, t_s)$. If $\mathcal{C} = (C, \dots, C)$ then we write $\|\mathbf{t}\|_{C^s,K}$ instead of $\|\mathbf{t}\|_{\mathcal{C},K}$. A question posed by V. Milman is to determine if, in the case $C = K$, one has that $\|\cdot\|_{K^s,K}$ is equivalent to the standard Euclidean norm up to a term which is logarithmic in the dimension, and in particular, if under some cotype condition on the norm induced by K to $\mathbb{R}^n$ one has equivalence between $\|\cdot\|_{K^s,K}$ and the Euclidean norm. Using rearrangement inequalities, Gluskin and V. Milman showed that if $\mathrm{vol}_n(C_j) = \mathrm{vol}_n(K)$ for all $1 \leqslant j \leqslant s$ then

$$\|\mathbf{t}\|_{\mathcal{C},K} \geqslant c\,\|\mathbf{t}\|_2$$

for all $\mathbf{t} \in \mathbb{R}^s$, where $c > 0$ is an absolute constant. We provide upper bounds for the multi-integral expression

$$\|\mathbf{t}\|_{C^s,K} = \int_C \cdots \int_C \Big\| \sum_{j=1}^s t_j x_j \Big\|_K dx_1 \cdots dx_s$$

in the case where C is isotropic. Our approach provides an alternative proof of the sharp lower bound, due to Gluskin and V. Milman, for this quantity.

This is a joint work with G. Chasapis and N. Skarmogiannis.

Bernardo Gonzalez Merino

University of Sevilla, Sevilla, Spain

Title: *On Hermite-Hadamard and Jensen inequalities.*

Abstract: The original Hermite-Hadamard inequality (1881-1893) states that if $f : \mathbb{R} \to \mathbb{R}$ is a concave function, then

$$\frac{1}{b-a}\int_a^b f(x)dx \leq f\left(\frac{a+b}{2}\right) \tag{1}$$

for any $a < b$. In this talk we will derive some new extensions of (1) in $\mathbb{R}^n$ replacing $f(x)$ by $f(x)^m$ for some $m \in \mathbb{N}$. As an application to these new inequalities, we will derive Rogers-Shephard type inequalities. The latter inequalities relate the volume of a convex compact set K in $\mathbb{R}^n$ to the volumes of some of its sections and projections with respect to some linear subspaces, and one of them is considered as the reverse to the Brunn-Minkowski inequality.

Anna Gusakova

Ruhr University Bochum, Bochum, Germany

Title: *Limiting theorems for Poisson-Delaunay tessellation.*

Abstract: Let η be a stationary Poisson point process on $\mathbb{R}^d$ with intensity $\gamma \in (0, \infty)$ and $\mathcal{D}$ be a collection of all Delaunay simplices corresponding to η, which is called the Poisson-Delaunay tessellation of $\mathbb{R}^d$. In this talk we consider random simplex Z_μ, $\mu > -2$ with distribution

$$\mathbf{Pr}^0_\mu(\cdot) = \frac{1}{\gamma_\mu}\,\mathbb{E}\sum_{\substack{c\in\mathcal{D}\\ z(c)\in[0,1]^d}} \mathbf{1}\{c - z(c) \in \cdot\}\, V_d(c)^{\mu+1},$$

where $V_d(c)$ and $z(c)$ stand for the volume and for the midpoint of the circumcenter of c respectively, and γ_μ is normalizing constant. Let us point out that Z_{-1} corresponds to typical Delaunay simplex (with respect to the circumcenter as centring function) and Z_0 coincides in distribution with the almost surely uniquely determined simplex from $\mathcal{D}$ containing the origin of $\mathbb{R}^d$. We investigate limiting behaviour of logarithmic volume of random simplex Z_μ when $d \to \infty$. Moreover, we consider different regimes when parameter μ and dimension d tend to infinity simultaneously.

Based on a joint work with Cristoph Thäle.

Thomas Hack

Vienna University of Technology, Vienna, Austria

Title: *Randomized Urysohn-type inequalities.*

Abstract: As a natural analog of Urysohn's inequality in Euclidean space, Gao, Hug, and Schneider showed in 2003 that in spherical or hyperbolic space, the total measure of totally geodesic hypersurfaces that meet a given convex body K is minimized when K is a geodesic ball. We present a random extension of this result by taking K to be the convex hull of finitely many points drawn according to a probability distribution and by showing that the minimum is attained at uniform distributions on geodesic balls. As a corollary we obtain a randomized Blaschke–Santaló inequality on the sphere.

This is a joint work with P. Pivovarov.

Georg Hofstätter

Vienna University of Technology, Vienna, Austria

Title: *Blaschke-Santaló inequalities for Minkowski Endomorphisms.*

Abstract: In this talk, we present a family of new isoperimetric inequalities for monotone Minkowski endomorphisms that interpolates between the Blaschke-Santaló and the Urysohn inequality. Among this large family of inequalities, the only affine invariant inequality turns out to be the strongest one. An extension of the family to merely weakly monotone Minkowski endomorphisms is shown to be impossible.

This is a joint work with F.E. Schuster.

Carlos Hugo Jiménez

PUC-Rio, Rio de Janeiro, Brazil

Title: *A functional version of the Busemann-Petty centroid inequality and its connections with Sobolev inequalities.*

Abstract: In this talk we will introduce a functional version of the L_p Busemann-Petty centroid inequality for convex bodies that is derived of some sort of functional mixed volume. We will next show some connections of this inequality with some Affine Sobolev type inequalities.

Boris Kashin

Steklov Mathematical Institute, Moscow, Russia

Title: *Some theorems on the restriction of operator to coordinate subspace.*

Philipp Kniefacz

Vienna University of Technology, Vienna, Austria

Title: *Sharp Sobolev inequalities via projection averages.*

Abstract: In this talk we present a family of sharp Sobolev-type inequalities obtained from averages of the length of i-dimensional projections of the gradient of a function. This family has both the classical Sobolev inequality (for $i = n$) and the affine Sobolev-Zhang inequality (for $i = 1$) as special cases as well as a recently obtained Sobolev inequality of Haberl and Schuster (for $i = n - 1$). Moreover, we identify the strongest member in our family of analytic inequalities which turns out to be the only affine invariant one among them.

This is a joint work with F.E. Schuster.

Aleksandr Kolesnikov

Higher School of Economics, Moscow, Russia

Title: *On transportation inequalities on the sphere and log Brunn-Minkowski problem.*

Abstract: We discuss the transportation problem on the unit sphere with a logarithmic cost function and its relation to the log-Minkowski problem.

Vitali Milman

Tel Aviv University, Tel Aviv, Israel

Title: *Flowers and Non-Linear Constructions in Convex Geometry.*

Part 1 is based on a joint work with Emanuel Milman and Liran Rotem, Part 2 is based on a joint work with Liran Rotem.

Sergii Myroshnychenko

University of Alberta, Edmonton, Canada

Title: *On recognizing convex bodies from their shadows.*

Abstract: We discuss a few tomography-type approaches to a unique determination of convex bodies. In particular, we answer the question by Kincses and Kurusa in the class of convex polytopes: let P and Q be two convex polytopes both contained in the interior of a Euclidean ball. We prove that $P = Q$, provided that their sight cones from any point on the boundary sphere are congruent. We also show analogous results for spherical projections and sections.

Marton Naszodi

Ecole polytechnique federale de Lausanne, Switzerland

Title: *Extensions of results on approximate John decomposition.*

Abstract: A John decomposition of the identity is a positive linear combination of rank one orthogonal projections which is equal to the identity operator I in $\mathbb{R}^n$. According to Rudelson's theorem, one may always select roughly $n \log n$ of the projections whose positive linear combination with proper weights approximates I well. Recently, Batson, Spielman and Srivastava managed to show that cn operators suffice. We discuss whether such approximation results extend to larger classes of matrices.

This is a joint work with Alexandr Polyanskii and Grigoriy Ivanov.

Piotr Nayar

University of Warsaw, Warsaw, Poland

Title: *Hadamard products and concentration of log-concave measures.*

Abstract: We prove optimal, up to a universal constant, comparison of weak and strong moments of random vectors in arbitrary n-dimensional normed spaces. As a consequence we show that any log-concave random vector in $\mathbb{R}^n$ satisfies optimal concentration (in the sense of Latała-Wojtaszczyk) with a constant $n^{5/12}$. We also mention applications of our result to the theory of p-summing operators on finite dimensional Banach spaces.

Based on a joint work with Rafał Latała.

Holger Rauhut

Hausdorff Center for Mathematics, Bonn, Germany

Title: *On the geometry of random polytopes generated by heavy-tailed random vectors.*

Abstract: We study centrally symmetric random polytopes generated by independent copies of possibly heavy-tailed random vectors X. We show that a deterministic set, namely the polar body of a so-called floating body naturally associated with X, is contained in the random polytope with high probability. Two previously known results in this direction are as follows: If X is a standard Gaussian vector, then a suitably scaled version of the Euclidean unit ball is contained in the generated random polytope with high probability, while if X is a Rademacher vector then the intersection of an ℓ_∞-ball with a scaled Euclidean ball is contained in the generated random polytope with high probability. Our main result substantially generalizes these statements and provides a unifying approach. In particular, we allow heavy-tailed random vectors and do not require independence of the entries of X. In this way, we recover and improve more recent results of the described type and discover new ones such as for the geometry of random polytopes generated by q-stable random variables including random vectors with independent Cauchy distributed entries. We outline consequences for noise-blind sparse recovery.

This is a joint work with Olivier Guédon, Felix Krahmer, Christian Kümmerle and Shahar Mendelson.

Oleg Reinov

St. Petersburg State University, Russia

Title: *On products of s-nuclear operators, $s \in (0, 1]$.*

Abstract: We discuss results in connection with a question of B. Mityagin: Is it true that a product of two nuclear operators in Banach spaces can be factored through a trace-class operator in a Hilbert space?

Giving new counterexamples to the question, we also present the following:

1) A product of s-nuclear and q-nuclear operators can be factored through an S_r-operator, where $1/r = 1/s + 1/q - 3/2$ and S_r is a Schatten-von Neumann class. An analogous theorem holds for the products of several nuclear operators.

2) Finite dimensional analogues of theorems mentioned in 1) are also valid. In this case, we consider factorizations through S_t-operators for arbitrary $t \in (0, +\infty]$.

3) We show that the results from 2) are optimal for the products of any number of s-nuclear operators.

4) By using examples from 3) and "summing" infinitely many finite rank operators, we obtain the sharpness of the theorems from 1).

Shlomo Reisner

Haifa University, Haifa, Israel

Title: *Ellipsoids are the only local maximizers of the volume product.*

Abstract: Using results about shadow systems and Steiner symmetrization, we prove Blaschke-Santaló inequality in a strengthened form, namely: the local maximizers of the volume product of convex bodies are actually the global maximizers, that is: ellipsoids.

This is a joint work with Mathieu Meyer.

Mark Rudelson

University of Michigan, Ann Arbor, MI, USA

Title: *Circular law for sparse random matrices.*

Abstract: Consider a sequence of $n \times n$ random matrices A_n whose entries are independent identically distributed random variables. The circular law asserts that the distribution of the eigenvalues of properly normalized matrices A_n converges to the uniform measure on the unit disc as n tends to infinity. We prove this law for sparse random matrices under the optimal sparsity assumption.

This is a joint work with Konstantin Tikhomirov.

Oleg Rusakov

Saint-Petersburg State University, Saint-Petersburg, Russia

Title: *Limits of special random walks in polygones and a class of stochastic fractals.*

Abstract: We consider a special random walk of a particle in a polygone and obtain, as a limit, cut off fractals in the polygon,which are described in terms of uniform distributions of probability on the corresponding fractals. Serpinski triangle is an example. Changing a parameter of the random walk we obtain in a limit fractal type distributions with a tight support on the polygon. Construction for the limit distributions forming is essentially based on a generalization of the Fibonacci numbers.

Dmitry Ryabogin

Kent State University, Kent, OH, USA

Title: *On a local version of the fifth Busemann-Petty problem.*

Abstract: In 1956, Busemann and Petty posed a series of questions about symmetric convex bodies, of which only the first one has been solved. Their fifth problem asks the following.

Let K be an origin symmetric convex body in the n-dimensional Euclidean space and let H_x be a hyperplane passing through the origin orthogonal to a unit direction x. Consider a hyperplane G parallel to H_x and supporting to K and let

$$C(K,x) = vol_{n-1}(K \cap H_x) dist(0,G).$$

If there exists a constant C such that for all directions x we have $C(K,x) = C$, does it follow that K is an ellipsoid?

Their eighth problem asks the following.

Let K be an origin symmetric convex body in the n-dimensional Euclidean space and let H_x be a hyperplane passing through the origin orthogonal to a unit direction x. If there exists a constant C such that for all directions x we have

$$f_K(x) = C vol_{n-1}(K \cap H_x),$$

does it follow that K is an ellipsoid? Here f_K is the curvature function of K, which is a reciprocal of the Gaussian curvature viewed as a function of the unit normal vector.

We give the affirmative answers to both problems for bodies in R^n, $n \geq 3$, that are sufficiently close to the Euclidean ball in the Banach-Mazur distance.

This is a joint work with Maria Alfonseca, Fedor Nazarov and Vlad Yaskin.

Shay Sadovsky

Tel Aviv University, Tel Aviv, Israel

Title: *Mixed volume inequalities on a special class of convex bodies.*

Abstract: Anti-blocking convex bodies, also known as convex corners, are a special class for which some geometric inequalities hold true, i.e. Mahler's conjecture, in the form of Saint-Raymond's inequality. In this talk I will try to illuminate the reasons for which this class of bodies is uniquely simple. Specifically, I will show that there is a decomposition for Minkowski differences of such bodies, and find formulae and bounds for certain mixed volumes of such bodies. As corollaries, I will show Godbersen's conjecture holds for this class, and attain a near bound for Mahler's conjecture on an associated family of bodies.

Evgenii Safronenko

Higher School of Economics, St. Petersburg, Russia

Title: *New extremal convex bodies for affine measures of symmetry.*

Abstract: This talk is devoted to affine measures of symmetry based on distances between two affine invariant points. In 2011 Meyer, Schütt and Werner provided an example of convex bodies with separated invariant points. After that Mordhorst constructed convex bodies with extremal distance between the centers of John and Lowner ellipsoids.

Using elementary properties of affine invariant points, we give a different construction of convex bodies which are extremal for the case when the measure of symmetry is defined by the centroid and either the center of John ellipsoid or the center of Lowner ellipsoid. We also obtain a similar result for the measure defined by the centers of John and Lowner ellipsoids using a different approach than the Mordhorst's one.

Christos Saraglou

University of Ioannina, Ioannina, Greece

Title: *Constant parts of a function via isotropicity of its sections.*

Abstract: We prove a local version of a recently established theorem by Myroshnychenko, Ryabogin and the second named author. More specifically, we show that if $f : \mathbb{S}^{n-1} \to \mathbb{R}$ is a bounded measurable function, U is an open subset of $\mathbb{S}^{n-1}$ and the restriction of f onto any great sphere perpendicular to U is isotropic, then f equals a constant almost everywhere in $U^{\perp} := \bigcup_{u\in U}(\mathbb{S}^{n-1} \cap u^{\perp})$. For the needs of our proof, we obtain a new generalization of a result from classical differential geometry, in the setting of convex hypersurfaces, that we believe is of independent interest.

Carsten Schütt

Christian Albrechts University, Kiel, Germany

Title: *Half space depth and floating body.*

Abstract: Surprising relations of the renown concept of the halfspace depth for multivariate data with some notions from convex and affine geometry are discovered. Data depth may be regarded as a measure of symmetry for random vectors. As such, it stands as a generalization of the measures of symmetry for convex sets, well studied in geometry. Further, it is shown that the level sets of the halfspace depth coincide with the convex floating bodies used in the definition of the affine surface area for convex bodies in Euclidean spaces. The new connections enable us to partially resolve some persistent open problems regarding theoretical properties of the depth.

Marta Strzelecka

University of Warsaw, Warsaw, Poland

Title: *Estimates of norms of log-concave random matrices with dependent entries.*

Abstract: In the talk we will focus on the mean value of the operator norm of random matrices $(X_{ij})_{i\leq m, j\leq n}$. In the case when entries are i.i.d., many results describe the asymptotic behaviour of these norms. Recently, Latała, van Handel, and Youssef considered the centred Gaussian matrices in the case of non-identically distributed entries (i.e. the weighted Gaussian matrices). They proved that then the mean of the operator norm $\|\cdot\|_{\ell_2\to\ell_2}$ is comparable (with a constant independent of the dimensions of the matrix) to the expected value of maximum Euclidean norm of rows and columns. Estimates for operator norms of weighted Gaussian matrices acting from ℓ_p^n to ℓ_q^m were investigated by Guédon, Hinrichs, Litvak, and Prochno. The authors obtained the analogues bound, which depends logarithmically on the dimensions. We will see how to generalize their approach into the case of a wide class of matrices with independent log-concave rows. We will not need the independence of entries within a row.

Kateryna Tatarko

University of Alberta, Edmonton, Canada

Title: *On the reverse isoperimetric problem.*

Abstract: The classical isoperimetric problem asks which domain, among all domains with a fixed surface area, has maximal volume. The question has a long and beautiful history and has been generalized to a variety of different settings. On the other hand, one can formulate the reverse isoperimetric problem: under which conditions can one minimize the volume among all domains of a given constraint.

In this talk we consider a class of λ-concave bodies in $\mathbb{R}^n$; that is, convex bodies with the property that each of their boundary points supports a tangent ball of radius $\frac{1}{\lambda}$ that lies locally (around the boundary point) inside the body. In this class, we solve a reverse isoperimetric problem: we show that the convex hull of two balls of radius $\frac{1}{\lambda}$ (a sausage body) is a unique volume minimizer among all λ-concave bodies of given surface area.

This is a joint work with Roman Chernov and Kostiantyn Drach.

Konstantin Tikhomirov

The Georgia Institute of Technology, Atlanta, GA, USA

Title: *Singularity of random Bernoulli matrices.*

Abstract: For each n, let M_n be a random $n\times n$ matrix with i.i.d. ± 1 (Bernoulli) entries. We show that

$$\mathbb{P}\{M_n \text{ is singular}\} = (1/2 + o(1))^n,$$

which settles an old problem.

Tomasz Tkocz

Carnegie Mellon University, Pittsburgh, USA

Title: *On a certain convexity property for sections of the cross-polytope.*

Abstract: I shall present a certain inequality for dilations of the cross-polytope and volume of their central sections (strong B-inequality for the cross-polytope and Lebesgue measure restricted to a subspace). This is motivated by the log-Brunn-Minkowski conjecture.

This is a joint work with P. Nayar.

Petros Valettas

University of Missouri, Columbia, MO, USA

Title: *Concentration and Convexity.*

Abstract: The concentration of measure phenomenon is a fundamental tool for the probabilistic study of high-dimensional structures. Nonetheless, there exist several key situations that it is insufficient to capture the right order of magnitude. After reviewing the reasons for suboptimal results, I will present new stronger forms of concentration and anti-concentration estimates, which are explained by convexity rather than isoperimetry. Time permitting I will discuss applications in asymptotic geometric analysis.

Ioann Vasilyev

PDMI, St. Petersburg, Russia and University Paris Est, Marne-la-Vallee, France

Title: *On the minimizing rectifiable G chains of dimension* 2 *and codimension* 1 *in finite dimensional Banach spaces.*

Abstract: In this talk we shall discuss the existence of rectifiable G chains of dimension 2 and of codimension 1 minimizing the Hausdorff mass in finite dimensional normed spaces. If time permits, I shall present two different approaches: the first one uses methods of H.Federer, and the second one is inspired by some ideas of the integral geometry. The talk is based on the following common paper with Thierry De Pauw: https://arxiv.org/pdf/1812.04520.pdf.

Vladislav Vysotsky

University of Sussex, Falmer, England and PDMI, St. Petersburg, Russia

Title: *A probabilistic look at the arithmetic-geometric mean inequality.*

Abstract: It was shown by E. Gluskin and V.D. Milman (2003) that the classical arithmetic-geometric mean inequality can be reversed (up to a multiplicative constant) with high probability, when applied to coordinates of a point chosen with respect to the surface unit measure on a high-dimensional Euclidean sphere. We will present two asymptotic refinements of this phenomenon in the more general setting of the surface probability measure on a high-dimensional ℓ_p-sphere, and also show that sampling the point according to the uniform distribution on the ball enclosed by the ℓ_p-sphere yields the same results. First, we prove a central limit theorem, which allows us to identify the precise constant in the reverse inequality. Second, we prove a large deviations principle and find the rate function explicitly.

This is a joint work with Z. Kabluchko and J. Prochno.

Elisabeth Werner

Case Western Reserve University, Cleveland, OH, USA

Title: *On the affine surface area.*

Abstract: Given a convex body K in $\mathbb{R}^n$, we study the quantity

$$AS(K) = \sup_{K' \subseteq K} as(K')$$

where $as(K')$ denotes the affine surface area of K', and the supremum is taken over all convex subsets of K. We study continuity properties of $AS(K)$ and give asymptotic estimates.

Based on joint work with Han Huang and Carsten Schuett.

Pierre Youssef

University Paris Diderot, Paris, France

Title: *On the spectrum of random graphs.*

Abstract: Understanding the distribution of the spectrum of a random matrix as the dimension grows is one of the main problems in random matrix theory. This includes, among others, the study of the limiting spectral distribution and the behavior at the boundary of the support of the limiting measure. It is known that the empirical spectral distribution of a square random matrix (resp. symmetric) with i.i.d centered entries with unit variance converges to the circular law (resp. semi-circular) as the dimension grows. In this talk, we are interested in the stability of these results and the behavior of the spectrum when the i.i.d assumption is relaxed. Random graphs provide models encapsulating sparsity and dependence. The talk will investigate: 1-The limiting spectral distribution of random regular graphs, 2-The behavior of the extreme eigenvalues/singular values and the spectral gap of random graphs.

Dmitry Zaporozhets

PDMI, St. Petersburg, Russia

Title: *Generalized Busemann inequality.*

Abstract: We will discuss a result that generalizes both the Busemann intersection inequality and the Busemann random simplex inequality.

Based on a joint work with Alexander Litvak.

Ning Zhang

Huazhong University of Science and Technology, Wuhan, Hubei, China

Title: *Topics on Milman's problem.*

Abstract: In this talk, I will present a preliminary report of a problem asked by Vitali Milman. This is if there are two convex bodies K and L such that $K + L = K^o + L^o$, is it true that $K = L^o$.

Artem Zvavitch

Kent State University, Kent, OH, USA

Title: *Volume product and Lipschitz-free Banach spaces.*

Abstract: In this talk we will discuss the geometric and extremal properties of the convex body $B_{\mathcal{F}(M)}$, which is the unit ball Lipschitz-free Banach space associated to a finite metric space M. In particular we discuss the extreme properties of the volume product $\mathcal{P}(M)$ of $B_{\mathcal{F}(M)}$ and its polar body $(B_{\mathcal{F}(M)})^\circ$, when the number of elements of M is fixed. We will show that if $\mathcal{P}(M)$ is maximal among all the metric spaces with the same number of points, then all triangle inequalities in M are strict and $B_{\mathcal{F}(M)}$ is simplicial. We also discuss metric spaces minimizing $\mathcal{P}(M)$.