Topics in Milman's problem

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a joint work with Han Huang

Asymptotic Geometric Analysis IV
EIMI, St. Petersburg

July 1, 2019

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Outline

Milman's problem

Problem

Does $K + L = K^{\circ} + L^{\circ}$ *implies* $K = L^{\circ}$*?*

Milman's problem

Problem

Does $K + L = K^\circ + L^\circ$ *implies* $K = L^\circ$*?*

Problem

Does $h_k - 1/\rho_K$ *unique determine a convex body* K*?*

Milman's problem

Theorem (Florentin and Dan, 19+)

Let K and L be convex bodies in $\mathbb{R}^n$ containing a unit ball. If $K + L = K^\circ + L^\circ$, then $K = L^\circ$.

Milman's problem for polytopes

Theorem (Huang and Zh., 19+)

Let K and L be convex polytopes in $\mathbb{R}^n$. If $K + L^\circ = K^\circ + L$, then $K = L$.

Idea of proof

- For the set $\{\theta \in S^n : \rho_K(\theta) > \rho_L(\theta)\}$, consider the Gauss map $F_L(\theta)$, where

$$h_K(F_L(\theta)) > h_L(F_L(\theta)) > \rho_L(F_L(\theta)) > \rho_K(F_L(\theta))$$

- For the set $\{\theta \in S^n : \rho_L(\theta) > \rho_K(\theta)\}$, consider the Gauss map $F_K(\theta)$, where

$$h_L(F_K(\theta)) > h_K(F_K(\theta)) > \rho_K(F_K(\theta)) > \rho_L(F_K(\theta))$$

Idea of proof

- Pick one simple connected open set Θ_0 of $\{\rho_K(\theta) > \rho_L(\theta)\}$ and set
$$\Theta_{i+1} = F_K(F_L(\Theta_i)).$$
- It is not so hard to prove $\Theta_i \cap \Theta_j = \emptyset$.
- So $\{\rho_K(\theta) > \rho_L(\theta)\}$ should have infinity seperate parts.

Milman's problem for convex body containing a unit ball

Theorem (Huang and Zh., 19+)

Let K and L be convex bodies in $\mathbb{R}^n$ containing a unit ball. If $K + L^\circ = K^\circ + L$, then $K = L$.

Idea of proof

- Pick one point $\theta_0 \in \{\rho_K(\theta) > \rho_L(\theta)\}$ and set

$$\theta_{i+1} = F_K(F_L(\theta_i)).$$

  This sequence convergences to point θ where

$$\rho_K(\theta) = h_K(\theta) = \rho_L(\theta) = h_L(\theta).$$

- At this point, it is clear $D^2(h_K(\theta)) = D^2(h_L(\theta))$, which means F_K and F_L convergence to this point asymptoticly.
- Consider the projection of difference between supporting points of K and L, which is never vanishing.

Current work

Theorem (Huang and Zh., 19+)

There exist two distinct convex bodies K and L with $K + L^\circ = K^\circ + L$.

Idea of proof

- Take the elliptic E with $\frac{x^2}{a^2} + a^2y^2 = 1$ $(a > 1)$.
- Pick one point $\theta_0 \in E$ and set $\theta_{i+1} = F_K(F_L(\theta_i))$ and $\Theta_{i+1} = [\theta_i, \theta_{i+1}]$.
- Set $\rho_K(\theta) = \rho_E(\theta) + \epsilon f_1$ in Θ_1, where $f_1'(\theta_0) = f_1'(\theta_1) = 0$.
- When θ_i close to $(0, \frac{1}{a})$, $|\rho_K - \rho_L| \leq c\frac{1}{a^i}$ and $|\rho_K' - \rho_L'| \leq c\frac{1}{a^i}$, hence $|(\rho_K^2 + 2(\rho_K')^2 - \rho_K\rho_K'') - (\rho_L^2 + 2(\rho_L')^2 - \rho_L\rho_L'')| \leq c\frac{1}{a^{2i}}$

Thank You!