MIDTERM. MATH 543.

INSTRUCTOR: A. Litvak **DATE and TIME:** 14:00 - 16:00, February 25, 2011.

INSTRUCTIONS: Explain your answers. Show your work for each question. Follow the regulations given in the examination booklet. Do not speak to or communicate with other students. Calculators, textbooks are not allowed.

1. Give the definitions of
a. (5 pt) a measurable set,
b. (8 pt) Lebesgue and Borel measurable function,
c. (5 pt) convergence in measure,
d. (5 pt) almost uniform convergence,
e. (5 pt) uniform convergence a.e.,
f. (8 pt) continuity from above and from below.

2. (8 pt) State the theorem which characterizes measures via continuity of functions on a ring.

3. (12 pt) Is the condition "measure is finite" in the Egoroff Theorem necessary? (Explain your answer).

4. (12 pt) Let λ be the Lebesgue measure on $\mathbb{R}$. Show that for every $\varepsilon > 0$ and every Lebesgue measurable set E there exists an open F such that $E \subset F$ and $\lambda(F \setminus E) < \varepsilon$ (you may use that $\lambda^*(A) = \inf\{\lambda(B) \ : \ A \subset B, \ B \text{ is open}\}$).

5. (20 pt) Let f be a function. Is it true that
a. f is measurable if and only if f^+ and f^- are measurable.
b. f is measurable if and only if $|f|$ is measurable.
Justify your answer.

6. (12 pt) Does convergence in measure imply pointwise a.e. convergence? (Explain your answer.)

7. (Bonus) Let μ be a finite measure on Borel subsets of $[0, 1]$ satisfying $\mu(\{x\}) = 0$ for every x. Show that for every $\varepsilon > 0$ there exists a dense open F such that $\mu(F) < \varepsilon$.