MIDTERM. MATH 543.

INSTRUCTOR: A. Litvak **DATE and TIME:** 14:00 - 16:00, October 31, 2008.

INSTRUCTIONS: Explain your answers. Show your work for each question. Follow the regulations given in the examination booklet. Do not speak to or communicate with other students. Calculators, textbooks are not allowed.

1. Give the definitions of
a. (4 pt) almost uniform convergence,
b. (4 pt) convergence in measure,
c. (4 pt) measurable function,
d. (4 pt) Lebesgue measurable function,
e. (4 pt) inner measure,
f. (4 pt) complete measure,
g. (4 pt) monotone class

2. (6 pt) State Egoroff Theorem.

3. (6 pt) Describe construction of the completion of a measure on a σ-ring.

4. (5 pt) Does almost uniform convergence imply convergence in measure? (Explain your answer).

5. (5 pt) Does pointwise a.e. convergence imply convergence in measure? (Explain your answer.)

6. (8 pt) Let $\mathcal{F}$ be a set of Lebesgue measurable functions. Let F be the function, defined by $F(x) = \sup\{f(x) \mid f \in \mathcal{F}\}$. Is F necessarily Lebesgue measurable?

7. (8 pt) Let $\{f_n\}_n$ be a sequence of measurable functions, convergent in measure to measurable functions f and g. Show that $f = g$ a.e.

8. (17 pt) Let $X = (X, S, \mu)$ be a measure space. Let $\{E_n\}_n$ be a sequence of measurable sets such that $\mu(\cup_{n\geq m} E_n) < \infty$ for some m. Show that

$$\mu\left(\limsup_{n\to\infty} E_n\right) \geq \limsup_{n\to\infty} \mu(E_n).$$

Is the condition "$\mu(\cup_{n\geq m} E_n) < \infty$ for some m" needed?

9. (17 pt) Let μ be a finite measure of X. Let $\{f_n\}_n$ be a sequence of finite measurable functions, convergent in measure to a finite measurable function f. Show that for every finite measurable function g the sequence $\{gf_n\}_n$ converges in measure to gf. Is the condition "μ is finite" needed?