Assignment # 5.

Due March 23

Problem 1. Let λ denote the Lebesgue measure on $\mathbb{R}$. Is it true that for every sequence of bounded integrable functions $\{f_n\}_{n\geq 1}$ from A to $\mathbb{R}$ one has

$$\int_A \liminf_{n\to\infty} f_n \; d\lambda \leq \liminf_{n\to\infty} \int_A f_n \; d\lambda \;,$$

where
a. $A = [0, 1]$?
b. $A = \mathbb{R}$ and the sequence is uniformly bounded?

Problem 2. Let (X, S, μ) be a measure space, $E \in S$, $\mu(E) < \infty$. Let $f : E \to \bar{\mathbb{R}}$ be a measurable function satisfying $\mu\{x \in E \mid |f(x)| > t\} \leq 1/t^2$ for every $t > 1$. Is f integrable? Is the condition "$\mu(E) < \infty$" important?

Problem 3. Let (X, S, μ) be a measure space. Let $f : X \to \bar{\mathbb{R}}$ be a measurable function such that

$$\nu(E) = \int_E f \; d\mu$$

is defined for every $E \in S$. Describe a Hahn decomposition with respect to ν.