Assignment # 4.

Due March 9

Problem 1. Let $X = (\mathbb{R}, \mu)$, where μ is the Lebesgue measure. Let $f : X \to \mathbb{R}$ be continuous a.e. (that is, $\mu(\{x \mid f \text{ is not continuous at } x\}) = 0$). Show that f is Lebesgue measurable.

Problem 2. Show that for every Lebesgue measurable function $f : \mathbb{R} \to \mathbb{R}$ there exists a Borel measurable function $g : \mathbb{R} \to \mathbb{R}$ such that $f = g$ a.e. with respect to the Lebesgue measure.

Problem 3. Let (X, S, μ) be a measure space such that $\mu(X) < \infty$. Let $\{f_n\}_n$ be a sequence of measurable functions which is convergent in measure to a measurable function f. Show that $\{f_n^2\}_n$ is convergent in measure to f^2.