Assignment # 2.

Due Feb. 16

Problem 1. Let μ_1^*, μ_2^* be finite outer measures on $\mathcal{P}(X)$. Define μ^* on $\mathcal{P}(X)$ by $\mu^* = \mu_1^* + \mu_2^*$. As usual, let $\bar{S}$, $\bar{S}_1$, and $\bar{S}_2$ denote the classes of measurable sets of μ^*, μ_1^*, and μ_2^* correspondingly. Show that $\bar{S} = \bar{S}_1 \cap \bar{S}_2$.

Problem 2. Let μ^* be an outer measure on a hereditary σ-ring H. Let E be a μ^*-measurable set and $F \in H$. Show that

$$\mu^*(E) + \mu^*(F) = \mu^*(E \cap F) + \mu^*(E \cup F).$$

Problem 3. Let $X = \mathbb{N}$ (the set of all positive integers). Define μ^* on $\mathcal{P}(X)$ by

$$\mu^*(E) = \limsup_{i \to \infty} \left(\frac{1}{n} \operatorname{card} \left(E \cap \{1, \ldots, n\} \right) \right).$$

Is μ^* an outer measure?