Assignment # 1.

Due Feb. 2

Problem 1. Let X be a set, $R \subset \mathcal{P}(X)$ be a σ-ring, μ be a measure on R. Show that the set

$$\{A \,|\, A \text{ is } \sigma\text{-finite}\}$$

is a σ-ring.

Problem 2. Let X be a set, $R \subset \mathcal{P}(X)$ be a ring, μ be a measure on R. Introduce a relation "$\sim$" on R by $E \sim F$ iff $\mu(E \Delta F) = 0$.
a. Show that $E \sim F$ implies $\mu(E) = \mu(F) = \mu(E \cap F)$.
b. Is $\{E \,|\, E \sim \emptyset\}$ a ring?

Problem 3. Let X be a set, $R \subset \mathcal{P}(X)$ be a ring, μ be a measure on R. Let $\{E_i\}_{i=1}^{\infty}$ be a sequence in R such that

$$\liminf_{i\to\infty} E_i \in R \quad \text{and for every } n \text{ one has } \bigcap_{i=n}^{\infty} E_i \in R.$$

Show that

$$\mu\left(\liminf_{i\to\infty} E_i\right) \leq \liminf_{i\to\infty} \mu(E_i).$$