

Additional Exercises.

Problem 1. Solve Problem 2 from Homework Assignment # 5 with the condition “ $\mu\{|f| > t\} \leq 1/t$ for every $t > 1$ ” instead of “ $\mu\{|f| > t\} \leq 1/t^2$ for every $t > 1$.”

Problem 2. Let $X = (X, S)$ be a measurable space, μ and ν be totally finite measures on X such that $\nu \ll \mu$. Let $\lambda = \mu + \nu$ and $f = \frac{d\nu}{d\lambda}$. Show that $0 \leq f \leq 1$ λ -a.e. and that $0 \leq f < 1$ μ -a.e.

Problem 3. Let $X = (X, S)$ be a measurable space, μ and ν be totally σ -finite measures on X such that $\nu \ll \mu$ and $\mu \ll \nu$. Let $\lambda = \mu + \nu$. Show that

$$\frac{d\nu}{d\mu} \frac{d\mu}{d\nu} = 1 \quad \lambda\text{- a.e.}$$