Additional Exercises.

Problem 1.

$$\text{Let } \ T = \{t_{ij}\}_{i,j \le n} : \ell_2^n \to \ell_2^n. \ \text{ Find } \ \|T\|_{\mathrm{HS}}.$$

Problem 2. Let $\{e_k\}_{k\ge1}$ be an o.n.b. of a Hilbert space H and $\{a_k\}_{k\ge1}$ be a sequence in $\mathbb{K}$. Define $T \in \mathcal{L}(H)$ by

$$Tx = \sum_{k\ge1} a_k (x, e_k) e_k.$$

Find conditions on $\{a_k\}_{k\ge1}$ such that
a. T is well-defined; **b.** $T \in \mathcal{B}(H)$; **c.** $T \in \mathcal{K}(H)$.

Problem 3. Let X be a complex Banach space and $T \in \mathcal{B}(H)$ be such that $\|Tx\| = \|x\|$ for every $x \in X$. Show that

$$\sigma(T) \subset \{\lambda \ \mid \ |\lambda| = 1\}.$$