Quiz # 5.

Problem 1. Find the unit tangent vector of $\vec{r}(t) = (2^t, \cos(3t), \ln(3t))$ at $t = 2$.

Solution.

$$\vec{r}\,'(t) = ((2^t)', (\cos(3t))', (\ln(3t))') = (2^t\ \ln 2, -3\sin(3t), 1/t).$$

Therefore, $\vec{r}(2) = (4\ln 2, -3\sin 6, 1/2)$ and the unit tangent vector is

$$\vec{T}(2) = \frac{\vec{r}\,'(2)}{|\vec{r}\,'(2)|} = \frac{(4\ln 2, -3\sin 6, 1/2)}{\sqrt{16\ln^2 2 + 9\sin^2 6 + 1/4}}.$$

Answer.

$$\vec{T}(2) = \frac{(4\ln 2, -3\sin 6, 1/2)}{\sqrt{16\ln^2 2 + 9\sin^2 6 + 1/4}}.$$

Problem 2. Find the arc length function of

$$\vec{r}(t) = (e^t, \sqrt{2}t, e^{-t}), \qquad 0 \le t \le 1.$$

Solution. We have $\vec{r}(t) = (f(t), g(t), h(t))$ for $f(t) = e^t$, $g(t) = \sqrt{2}t$, $h(t) = e^{-t}$. Then

$$(f'(t))^2 + (g'(t))^2 + (h'(t))^2 = (e^t)^2 + (\sqrt{2})^2 + (e^{-t})^2 = (e^t)^2 + 2e^te^{-t} + (e^{-t})^2 = (e^t + e^{-t})^2.$$

Therefore the arc length function on $[0, 1]$ is

$$s(t) = \int_0^t \sqrt{(f'(u))^2 + (g'(u))^2 + (h'(u))^2}\ du = \int_0^t (e^u + e^{-u})du = (e^u - e^{-u})\Big|_0^t = e^t - e^{-t}.$$

Answer. $s(t) = e^t - e^{-t}$, $0 \le t \le 1$.

Problem 3. Find the domain of $f(x, y) = \ln(y^3 - x)$.

Solution. The expression is defined whenever $y^3 - x > 0$, therefore the domain is the set of all points (x, y) such that $x < y^3$.

Answer. $\{(x, y) \mid x < y^3\}$.

Remark. Note that just equation $x > y^3$ is not acceptable as an answer, since the domain is a set of points on the plane (equivalently, a set of two-dimensional vectors) and not an equation.