# Quiz # 5.

**Problem 1.** Let $\vec{r}(t) = (\cos t, \ln t, e^{2t})$ be a vector function.
**a.** Find the derivative of $\vec{r}(t)$. **b.** Find the unit tangent vector of $\vec{r}(t)$ at $t = 1$.

**Solution.** The derivative is

$$\vec{r}\,'(t) = ((\cos t)', (\ln t)', (e^{2t})') = (-\sin t, 1/t, 2e^{2t}).$$

Therefore, $\vec{r}(1) = (-\sin 1, 1, 2e^2)$ and the unit tangent vector is

$$\vec{T}(1) = \frac{\vec{r}\,'(1)}{|\vec{r}\,'(1)|} = \frac{(-\sin 1, 1, 2e^2)}{\sqrt{\sin^2 1 + 1 + 4e^4}}.$$

**Answer.**

$$\vec{r}\,'(t) = (-\sin t, 1/t, 2e^{2t}), \qquad \vec{T}(1) = \frac{(-\sin 1, 1, 2e^2)}{\sqrt{1 + \sin^2 1 + 4e^4}}.$$

**Problem 2.** Find the length of the curve

$$\vec{r}(t) = (1/3)t^3\ \mathbf{i} - t^2\ \mathbf{j} + 2t\ \mathbf{k}, \quad 0 \le t \le 3.$$

**Solution.** We have $\vec{r}(t) = (f(t), g(t), h(t))$ for $f(t) = (1/3)t^3$, $g(t) = -t^2$, $h(t) = 2t$. The length is

$$\int_0^3 \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\ dt = \int_0^3 \sqrt{t^4 + 4t^2 + 4}\ dt = \int_0^3 \sqrt{(t^2+2)^2}\ dt$$

$$= \int_0^3 (t^2 + 2)\ dt = (t^3/3 + 2t)\Big|_0^3 = 15.$$

**Answer.** 15

**Problem 3.** Find the domain of $f(x, y) = \sqrt{x - y^2}$.

**Solution.** The expression is defined whenever $x - y^2 \ge 0$, therefore the domain is the set of all points $(x, y)$ such that $x \ge y^2$.

**Answer.** $\{(x, y)\ |\ x \ge y^2\}$.

**Remark.** Note that just equation $x \ge y^2$ is not acceptable as an answer, since the domain is a set of points on the plane (equivalentely, a set of two-dimensional vectors) and not an equation.