Quiz # 1.

Problem 1. Is the following sequence convergent? If yes, find the limit.

$$\left\{\frac{2n^3 - n^2 + 1}{n^4 + n^2 - 1}\right\}_{n=1}^{\infty}$$

Solution.

$$\lim_{n\to\infty} \frac{2n^3 - n^2 + 1}{n^4 + n^2 - 1} = \lim_{n\to\infty} \frac{2n^3/n^4 - n^2/n^4 + 1/n^4}{n^4/n^4 + n^2/n^4 - 1/n^4} = \lim_{n\to\infty} \frac{2/n - 1/n^2 + 1/n^4}{1 + 1/n^2 - 1/n^4} = \frac{0 - 0 + 0}{1 + 0 - 0} = 0$$

Answer. The sequence is convergent and its limit is 0.

Problem 2. Is the following series convergent? If yes, find its sum.

$$\sum_{n=1}^{\infty} (-5)^n \, 3^{-2n+1}$$

Solution. We have

$$\sum_{n=1}^{\infty} (-5)^n \, 3^{-2n+1} = \sum_{n=1}^{\infty} \frac{(-5)^n \cdot 3}{9^n} = \sum_{n=1}^{\infty} \frac{-5}{3} \, \frac{(-5)^{n-1}}{9^{n-1}} = \sum_{n=1}^{\infty} \frac{-5}{3} \left(\frac{-5}{9}\right)^{n-1}.$$

Since $|-5/9| < 1$, the series is convergent and

$$\sum_{n=1}^{\infty} \frac{-5}{3} \left(\frac{-5}{9}\right)^{n-1} = \frac{-5}{3} \, \frac{1}{1 + 5/9} = \frac{-5}{3} \, \frac{9}{14} = -\frac{15}{14}.$$

Answer. The series is convergent and its sum is $-1\frac{1}{14}$.