Quiz # 1.

Problem 1. Test the series for convergence or divergence. Find the sum whenever is possible.

a. $\displaystyle\sum_{n=0}^{\infty} \cos\left(\frac{\pi\, n^2}{6n^2-3n+2}\right)$ **b.** $\displaystyle\sum_{n=0}^{\infty} 2^{-3n-1}\, 7^{n+1}$ **c.** $\displaystyle\sum_{n=0}^{\infty} 3^{2n-1}\, e^{-2n+3}.$

Solution. **a.**

$$\lim_{n\to\infty} \cos\left(\frac{\pi\, n^2}{6n^2-3n+2}\right) = \cos\left(\lim_{n\to\infty} \frac{\pi\, n^2}{6n^2-3n+2}\right)$$
$$= \cos\left(\lim_{n\to\infty} \frac{\pi}{6-3/n+2/n^2}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \neq 0,$$

therefore by the divergence test the series is divergent.

b. Note

$$2^{-3n-1}\, 7^{n+1} = \frac{7\cdot 7^n}{2\cdot 2^{3n}} = \frac{7}{2}\left(\frac{7}{8}\right)^n.$$

As $7/8 < 1$ the series (which is a geometric series) is convergent and

$$\sum_{n=0}^{\infty} 2^{-3n-1}\, 7^{n+1} = \frac{7}{2}\sum_{n=0}^{\infty}\left(\frac{7}{8}\right)^n = \frac{7}{2}\,\frac{1}{1-7/8} = 28.$$

c. Note that

$$\lim_{n\to\infty} 3^{2n-1}\, e^{-2n+3} = \lim_{n\to\infty}\frac{e^3\, 3^{2n}}{3\, e^{2n}} = \lim_{n\to\infty}\frac{e^3}{3}\left(\frac{3}{e}\right)^{2n} = \infty,$$

since $r^n \to \infty$ as $n \to \infty$ for $r = (3/e)^2 > 1$. Therefore, by the divergence test, the series is divergent.

Another way to see that the series is divergent is to realize that it is a geometric series with $q = (3/e)^2 > 1$.

Answer. The series **a** and **c** are divergent, the series **b** is convergent and its sum is 28.