

12.5 Exercises

1. Determine whether each statement is true or false.
 - (a) Two lines parallel to a third line are parallel.
 - (b) Two lines perpendicular to a third line are parallel.
 - (c) Two planes parallel to a third plane are parallel.
 - (d) Two planes perpendicular to a third plane are parallel.
 - (e) Two lines parallel to a plane are parallel.
 - (f) Two lines perpendicular to a plane are parallel.
 - (g) Two planes parallel to a line are parallel.
 - (h) Two planes perpendicular to a line are parallel.
 - (i) Two planes either intersect or are parallel.
 - (j) Two lines either intersect or are parallel.
 - (k) A plane and a line either intersect or are parallel.
- 2–5 Find a vector equation and parametric equations for the line.
 2. The line through the point $(6, -5, 2)$ and parallel to the vector $\langle 1, 3, -\frac{2}{3} \rangle$
 3. The line through the point $(2, 2.4, 3.5)$ and parallel to the vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$
 4. The line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2t, y = 6 - 3t, z = 3 + 9t$
 5. The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$
- 6–12 Find parametric equations and symmetric equations for the line.
 6. The line through the origin and the point $(4, 3, -1)$
 7. The line through the points $(0, \frac{1}{2}, 1)$ and $(2, 1, -3)$
 8. The line through the points $(1.0, 2.4, 4.6)$ and $(2.6, 1.2, 0.3)$
 9. The line through the points $(-8, 1, 4)$ and $(3, -2, 4)$
 10. The line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$
 11. The line through $(1, -1, 1)$ and parallel to the line $x + 2 = \frac{1}{2}y = z - 3$
 12. The line of intersection of the planes $x + 2y + 3z = 1$ and $x - y + z = 1$
13. Is the line through $(-4, -6, 1)$ and $(-2, 0, -3)$ parallel to the line through $(10, 18, 4)$ and $(5, 3, 14)$?
14. Is the line through $(-2, 4, 0)$ and $(1, 1, 1)$ perpendicular to the line through $(2, 3, 4)$ and $(3, -1, -8)$?
15. (a) Find symmetric equations for the line that passes through the point $(1, -5, 6)$ and is parallel to the vector $\langle -1, 2, -3 \rangle$.
 (b) Find the points in which the required line in part (a) intersects the coordinate planes.
16. (a) Find parametric equations for the line through $(2, 4, 6)$ that is perpendicular to the plane $x - y + 3z = 7$.
 (b) In what points does this line intersect the coordinate planes?
17. Find a vector equation for the line segment from $(2, -1, 4)$ to $(4, 6, 1)$.
18. Find parametric equations for the line segment from $(10, 3, 1)$ to $(5, 6, -3)$.
- 19–22 Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.
 19. $L_1: x = 3 + 2t, y = 4 - t, z = 1 + 3t$
 $L_2: x = 1 + 4s, y = 3 - 2s, z = 4 + 5s$
 20. $L_1: x = 5 - 12t, y = 3 + 9t, z = 1 - 3t$
 $L_2: x = 3 + 8s, y = -6s, z = 7 + 2s$
 21. $L_1: \frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-1}{-3}$
 $L_2: \frac{x-3}{1} = \frac{y+4}{3} = \frac{z-2}{-7}$
 22. $L_1: \frac{x}{1} = \frac{y-1}{-1} = \frac{z-2}{3}$
 $L_2: \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7}$
- 23–40 Find an equation of the plane.
 23. The plane through the origin and perpendicular to the vector $\langle 1, -2, 5 \rangle$
 24. The plane through the point $(5, 3, 5)$ and with normal vector $2\mathbf{i} + \mathbf{j} - \mathbf{k}$
 25. The plane through the point $(-1, \frac{1}{2}, 3)$ and with normal vector $\mathbf{i} + 4\mathbf{j} + \mathbf{k}$
 26. The plane through the point $(2, 0, 1)$ and perpendicular to the line $x = 3t, y = 2 - t, z = 3 + 4t$
 27. The plane through the point $(1, -1, -1)$ and parallel to the plane $5x - y - z = 6$
 28. The plane through the point $(2, 4, 6)$ and parallel to the plane $z = x + y$
 29. The plane through the point $(1, \frac{1}{2}, \frac{1}{3})$ and parallel to the plane $x + y + z = 0$
 30. The plane that contains the line $x = 1 + t, y = 2 - t, z = 4 - 3t$ and is parallel to the plane $5x + 2y + z = 1$
 31. The plane through the points $(0, 1, 1), (1, 0, 1),$ and $(1, 1, 0)$
 32. The plane through the origin and the points $(2, -4, 6)$ and $(5, 1, 3)$

33. The plane through the points $(3, -1, 2)$, $(8, 2, 4)$, and $(-1, -2, -3)$
34. The plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$
35. The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$
36. The plane that passes through the point $(1, -1, 1)$ and contains the line with symmetric equations $x = 2y = 3z$
37. The plane that passes through the point $(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$
38. The plane that passes through the points $(0, -2, 5)$ and $(-1, 3, 1)$ and is perpendicular to the plane $2z = 5x + 4y$
39. The plane that passes through the point $(1, 5, 1)$ and is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$
40. The plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and is perpendicular to the plane $x + y - 2z = 1$

41–44 Use intercepts to help sketch the plane.

41. $2x + 5y + z = 10$

42. $3x + y + 2z = 6$

43. $6x - 3y + 4z = 6$

44. $6x + 5y - 3z = 15$

45–47 Find the point at which the line intersects the given plane.

45. $x = 3 - t$, $y = 2 + t$, $z = 5t$; $x - y + 2z = 9$

46. $x = 1 + 2t$, $y = 4t$, $z = 2 - 3t$; $x + 2y - z + 1 = 0$

47. $x = y - 1 = 2z$; $4x - y + 3z = 8$

48. Where does the line through $(1, 0, 1)$ and $(4, -2, 2)$ intersect the plane $x + y + z = 6$?

49. Find direction numbers for the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$.

50. Find the cosine of the angle between the planes $x + y + z = 0$ and $x + 2y + 3z = 1$.

51–56 Determine whether the planes are parallel, perpendicular, or neither. If neither, find the angle between them.

51. $x + 4y - 3z = 1$, $-3x + 6y + 7z = 0$

52. $2z = 4y - x$, $3x - 12y + 6z = 1$

53. $x + y + z = 1$, $x - y + z = 1$

54. $2x - 3y + 4z = 5$, $x + 6y + 4z = 3$

55. $x = 4y - 2z$, $8y = 1 + 2x + 4z$

56. $x + 2y + 2z = 1$, $2x - y + 2z = 1$

57–58 (a) Find parametric equations for the line of intersection of the planes and (b) find the angle between the planes.

57. $x + y + z = 1$, $x + 2y + 2z = 1$

58. $3x - 2y + z = 1$, $2x + y - 3z = 3$

59–60 Find symmetric equations for the line of intersection of the planes.

59. $5x - 2y - 2z = 1$, $4x + y + z = 6$

60. $z = 2x - y - 5$, $z = 4x + 3y - 5$

61. Find an equation for the plane consisting of all points that are equidistant from the points $(1, 0, -2)$ and $(3, 4, 0)$.

62. Find an equation for the plane consisting of all points that are equidistant from the points $(2, 5, 5)$ and $(-6, 3, 1)$.

63. Find an equation of the plane with x -intercept a , y -intercept b , and z -intercept c .

64. (a) Find the point at which the given lines intersect:

$$\mathbf{r} = \langle 1, 1, 0 \rangle + t\langle 1, -1, 2 \rangle$$

$$\mathbf{r} = \langle 2, 0, 2 \rangle + s\langle -1, 1, 0 \rangle$$

(b) Find an equation of the plane that contains these lines.

65. Find parametric equations for the line through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$.

66. Find parametric equations for the line through the point $(0, 1, 2)$ that is perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$ and intersects this line.

67. Which of the following four planes are parallel? Are any of them identical?

$$P_1: 3x + 6y - 3z = 6$$

$$P_2: 4x - 12y + 8z = 5$$

$$P_3: 9y = 1 + 3x + 6z$$

$$P_4: z = x + 2y - 2$$

68. Which of the following four lines are parallel? Are any of them identical?

$$L_1: x = 1 + 6t, \quad y = 1 - 3t, \quad z = 12t + 5$$

$$L_2: x = 1 + 2t, \quad y = t, \quad z = 1 + 4t$$

$$L_3: 2x - 2 = 4 - 4y = z + 1$$

$$L_4: \mathbf{r} = \langle 3, 1, 5 \rangle + t\langle 4, 2, 8 \rangle$$

69–70 Use the formula in Exercise 45 in Section 12.4 to find the distance from the point to the given line.

69. $(4, 1, -2)$; $x = 1 + t$, $y = 3 - 2t$, $z = 4 - 3t$

70. $(0, 1, 3)$; $x = 2t$, $y = 6 - 2t$, $z = 3 + t$