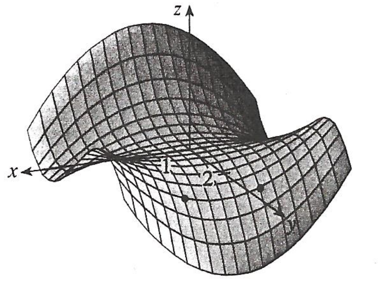
5–8 Determine the signs of the partial derivatives for the function f whose graph is shown.



5. (a) $f_x(1, 2)$ (b) $f_y(1, 2)$

6. (a) $f_x(-1, 2)$ (b) $f_y(-1, 2)$

7. (a) $f_{xx}(-1, 2)$ (b) $f_{yy}(-1, 2)$

8. (a) $f_{xy}(1, 2)$ (b) $f_{xy}(-1, 2)$

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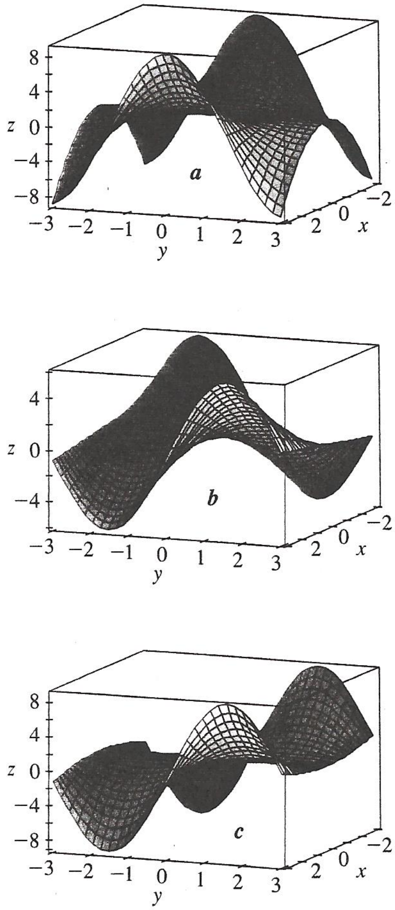
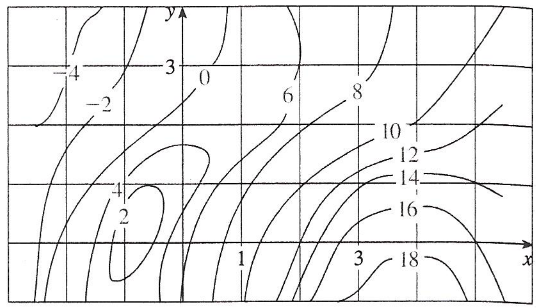
9. The following surfaces, labeled a, b, and c, are graphs of a function f and its partial derivatives f_x and f_y. Identify each surface and give reasons for your choices.



10. A contour map is given for a function f. Use it to estimate $f_x(2, 1)$ and $f_y(2, 1)$.



11. If $f(x, y) = 16 - 4x^2 - y^2$, find $f_x(1, 2)$ and $f_y(1, 2)$ and interpret these numbers as slopes. Illustrate with either hand-drawn sketches or computer plots.

12. If $f(x, y) = \sqrt{4 - x^2 - 4y^2}$, find $f_x(1, 0)$ and $f_y(1, 0)$ and interpret these numbers as slopes. Illustrate with either hand-drawn sketches or computer plots.

13–14 Find f_x and f_y and graph f, f_x, and f_y with domains and viewpoints that enable you to see the relationships between them.

13. $f(x, y) = x^2y^3$ **14.** $f(x, y) = \dfrac{y}{1 + x^2y^2}$

15–40 Find the first partial derivatives of the function.

15. $f(x, y) = y^5 - 3xy$ **16.** $f(x, y) = x^4y^3 + 8x^2y$

17. $f(x, t) = e^{-t}\cos \pi x$ **18.** $f(x, t) = \sqrt{x}\,\ln t$

19. $z = (2x + 3y)^{10}$ **20.** $z = \tan xy$

21. $f(x, y) = \dfrac{x}{y}$ **22.** $f(x, y) = \dfrac{x}{(x + y)^2}$

23. $f(x, y) = \dfrac{ax + by}{cx + dy}$ **24.** $w = \dfrac{e^v}{u + v^2}$

25. $g(u, v) = (u^2v - v^3)^5$ **26.** $u(r, \theta) = \sin(r\cos\theta)$

27. $R(p, q) = \tan^{-1}(pq^2)$ **28.** $f(x, y) = x^y$

29. $F(x, y) = \int_y^x \cos(e^t)\,dt$ **30.** $F(\alpha, \beta) = \int_\alpha^\beta \sqrt{t^3 + 1}\,dt$

31. $f(x, y, z) = xz - 5x^2y^3z^4$ **32.** $f(x, y, z) = x\sin(y - z)$

33. $w = \ln(x + 2y + 3z)$ **34.** $w = ze^{xyz}$

35. $u = xy\sin^{-1}(yz)$ **36.** $u = x^{y/z}$

37. $h(x, y, z, t) = x^2y\cos(z/t)$ **38.** $\phi(x, y, z, t) = \dfrac{\alpha x + \beta y^2}{\gamma z + \delta t^2}$

39. $u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

40. $u = \sin(x_1 + 2x_2 + \cdots + nx_n)$

41–44 Find the indicated partial derivative.

41. $f(x, y) = \ln\left(x + \sqrt{x^2 + y^2}\right)$; $f_x(3, 4)$

42. $f(x, y) = \arctan(y/x)$; $f_x(2, 3)$

43. $f(x, y, z) = \dfrac{y}{x + y + z}$; $f_y(2, 1, -1)$

44. $f(x, y, z) = \sqrt{\sin^2 x + \sin^2 y + \sin^2 z}$; $f_z(0, 0, \pi/4)$

45– Use the definition of partial derivatives as limits [4] to find $f_x(x, y)$ and $f_y(x, y)$.

45. $f(x, y) = xy^2 - x^3 y$

46. $f(x, y) = \dfrac{x}{x + y^2}$

47– Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

47. $x^2 + 2y^2 + 3z^2 = 1$

48. $x^2 - y^2 + z^2 - 2z = 4$

49. $e^z = xyz$

50. $yz + x \ln y = z^2$

51– Find $\partial z/\partial x$ and $\partial z/\partial y$.

51. (a) $z = f(x) + g(y)$ (b) $z = f(x + y)$

52. (a) $z = f(x)g(y)$ (b) $z = f(xy)$
(c) $z = f(x/y)$

53– Find all the second partial derivatives.

53. $f(x, y) = x^3 y^5 + 2x^4 y$

54. $f(x, y) = \sin^2(mx + ny)$

55. $w = \sqrt{u^2 + v^2}$

56. $v = \dfrac{xy}{x - y}$

57. $z = \arctan \dfrac{x + y}{1 - xy}$

58. $v = e^{xe^y}$

59– Verify that the conclusion of Clairaut's Theorem holds, that is, $u_{xy} = u_{yx}$.

59. $u = x^4 y^3 - y^4$

60. $u = e^{xy} \sin y$

61. $u = \cos(x^2 y)$

62. $u = \ln(x + 2y)$

63– Find the indicated partial derivative(s).

63. $f(x, y) = x^4 y^2 - x^3 y$; f_{xxx}, f_{xyx}

64. $f(x, y) = \sin(2x + 5y)$; f_{yxy}

65. $f(x, y, z) = e^{xyz^2}$; f_{xyz}

66. $g(r, s, t) = e^r \sin(st)$; g_{rst}

67. $u = e^{r\theta} \sin \theta$; $\dfrac{\partial^3 u}{\partial r^2 \, \partial \theta}$

68. $z = u\sqrt{v - w}$; $\dfrac{\partial^3 z}{\partial u \, \partial v \, \partial w}$

69. $w = \dfrac{x}{y + 2z}$; $\dfrac{\partial^3 w}{\partial z \, \partial y \, \partial x}$, $\dfrac{\partial^3 w}{\partial x^2 \, \partial y}$

70. $u = x^a y^b z^c$; $\dfrac{\partial^6 u}{\partial x \, \partial y^2 \, \partial z^3}$

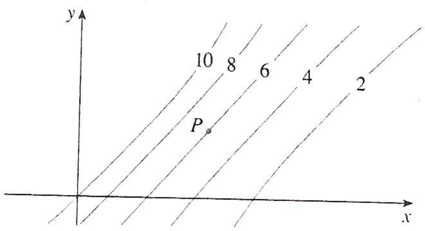
71. If $f(x, y, z) = xy^2 z^3 + \arcsin(x\sqrt{z})$, find f_{xzy}. [*Hint:* Which order of differentiation is easiest?]

72. If $g(x, y, z) = \sqrt{1 + xz} + \sqrt{1 - xy}$, find g_{xyz}. [*Hint:* Use a different order of differentiation for each term.]

73. Use the table of values of $f(x, y)$ to estimate the values of $f_x(3, 2)$, $f_x(3, 2.2)$, and $f_{xy}(3, 2)$.

x \ y	1.8	2.0	2.2
2.5	12.5	10.2	9.3
3.0	18.1	17.5	15.9
3.5	20.0	22.4	26.1

74. Level curves are shown for a function f. Determine whether the following partial derivatives are positive or negative at the point P.
(a) f_x (b) f_y (c) f_{xx}
(d) f_{xy} (e) f_{yy}



75. Verify that the function $u = e^{-\alpha^2 k^2 t} \sin kx$ is a solution of the *heat conduction equation* $u_t = \alpha^2 u_{xx}$.

76. Determine whether each of the following functions is a solution of Laplace's equation $u_{xx} + u_{yy} = 0$.
(a) $u = x^2 + y^2$ (b) $u = x^2 - y^2$
(c) $u = x^3 + 3xy^2$ (d) $u = \ln \sqrt{x^2 + y^2}$
(e) $u = \sin x \cosh y + \cos x \sinh y$
(f) $u = e^{-x} \cos y - e^{-y} \cos x$

77. Verify that the function $u = 1/\sqrt{x^2 + y^2 + z^2}$ is a solution of the three-dimensional Laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$.

78. Show that each of the following functions is a solution of the wave equation $u_{tt} = a^2 u_{xx}$.
(a) $u = \sin(kx)\sin(akt)$ (b) $u = t/(a^2 t^2 - x^2)$
(c) $u = (x - at)^6 + (x + at)^6$
(d) $u = \sin(x - at) + \ln(x + at)$

79. If f and g are twice differentiable functions of a single variable, show that the function

$$u(x, t) = f(x + at) + g(x - at)$$

is a solution of the wave equation given in Exercise 78.