

is a rational function of three variables and so is continuous at every point in \mathbb{R}^3 except where $x^2 + y^2 + z^2 = 1$. In other words, it is discontinuous on the sphere with center the origin and radius 1.

If we use the vector notation introduced at the end of Section 14.1, then we can write the definitions of a limit for functions of two or three variables in a single compact form as follows.

5 If f is defined on a subset D of \mathbb{R}^n , then $\lim_{\mathbf{x} \rightarrow \mathbf{a}} f(\mathbf{x}) = L$ means that for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that

$$\text{if } \mathbf{x} \in D \text{ and } 0 < |\mathbf{x} - \mathbf{a}| < \delta \text{ then } |f(\mathbf{x}) - L| < \varepsilon$$

Notice that if $n = 1$, then $\mathbf{x} = x$ and $\mathbf{a} = a$, and **5** is just the definition of a limit for functions of a single variable. For the case $n = 2$, we have $\mathbf{x} = \langle x, y \rangle$, $\mathbf{a} = \langle a, b \rangle$, and $|\mathbf{x} - \mathbf{a}| = \sqrt{(x - a)^2 + (y - b)^2}$, so **5** becomes Definition 1. If $n = 3$, then $\mathbf{x} = \langle x, y, z \rangle$, $\mathbf{a} = \langle a, b, c \rangle$, and **5** becomes the definition of a limit of a function of three variables. In each case the definition of continuity can be written as

$$\lim_{\mathbf{x} \rightarrow \mathbf{a}} f(\mathbf{x}) = f(\mathbf{a})$$

14.2 Exercises

- Suppose that $\lim_{(x,y) \rightarrow (3,1)} f(x,y) = 6$. What can you say about the value of $f(3,1)$? What if f is continuous?
- Explain why each function is continuous or discontinuous.
 - The outdoor temperature as a function of longitude, latitude, and time
 - Elevation (height above sea level) as a function of longitude, latitude, and time
 - The cost of a taxi ride as a function of distance traveled and time

3–4 Use a table of numerical values of $f(x,y)$ for (x,y) near the origin to make a conjecture about the value of the limit of $f(x,y)$ as $(x,y) \rightarrow (0,0)$. Then explain why your guess is correct.

$$3. f(x,y) = \frac{x^2y^3 + x^3y^2 - 5}{2 - xy}$$

$$4. f(x,y) = \frac{2xy}{x^2 + 2y^2}$$

5–22 Find the limit, if it exists, or show that the limit does not exist.

$$5. \lim_{(x,y) \rightarrow (1,2)} (5x^3 - x^2y^2)$$

$$6. \lim_{(x,y) \rightarrow (1,-1)} e^{-xy} \cos(x+y)$$

$$7. \lim_{(x,y) \rightarrow (2,1)} \frac{4 - xy}{x^2 + 3y^2}$$

$$8. \lim_{(x,y) \rightarrow (1,0)} \ln\left(\frac{1 + y^2}{x^2 + xy}\right)$$

$$9. \lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

$$10. \lim_{(x,y) \rightarrow (0,0)} \frac{5y^4 \cos^2 x}{x^4 + y^4}$$

$$11. \lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$$

$$12. \lim_{(x,y) \rightarrow (1,0)} \frac{xy - y}{(x-1)^2 + y^2}$$

$$13. \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$$

$$14. \lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^4}{x^2 + y^2}$$

$$15. \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y e^y}{x^4 + 4y^2}$$

$$16. \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2}$$

$$17. \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2} + 1 - 1}$$


$$18. \lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2 + y^8}$$

$$19. \lim_{(x,y,z) \rightarrow (\pi, 0, 1/3)} e^{y^3} \tan(xz)$$

$$20. \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2}$$

$$21. \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$$

$$22. \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{yz}{x^2 + 4y^2 + 9z^2}$$

 23–24 Use a computer graph of the function to explain why the limit does not exist.

$$23. \lim_{(x,y) \rightarrow (0,0)} \frac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$$

$$24. \lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}$$