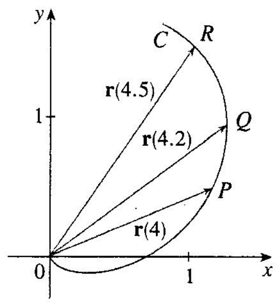
13.2 Exercises

1. The figure shows a curve C given by a vector function $\mathbf{r}(t)$.
   (a) Draw the vectors $\mathbf{r}(4.5) - \mathbf{r}(4)$ and $\mathbf{r}(4.2) - \mathbf{r}(4)$.
   (b) Draw the vectors
   $$\frac{\mathbf{r}(4.5) - \mathbf{r}(4)}{0.5} \quad \text{and} \quad \frac{\mathbf{r}(4.2) - \mathbf{r}(4)}{0.2}$$
   (c) Write expressions for $\mathbf{r}'(4)$ and the unit tangent vector $\mathbf{T}(4)$.
   (d) Draw the vector $\mathbf{T}(4)$.



2. (a) Make a large sketch of the curve described by the vector function $\mathbf{r}(t) = \langle t^2, t \rangle$, $0 \leqslant t \leqslant 2$, and draw the vectors $\mathbf{r}(1)$, $\mathbf{r}(1.1)$, and $\mathbf{r}(1.1) - \mathbf{r}(1)$.
   (b) Draw the vector $\mathbf{r}'(1)$ starting at $(1, 1)$, and compare it with the vector
   $$\frac{\mathbf{r}(1.1) - \mathbf{r}(1)}{0.1}$$
   Explain why these vectors are so close to each other in length and direction.

3–8
(a) Sketch the plane curve with the given vector equation.
(b) Find $\mathbf{r}'(t)$.
(c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of t.

3. $\mathbf{r}(t) = \langle t - 2, t^2 + 1 \rangle$, $t = -1$

4. $\mathbf{r}(t) = \langle t^2, t^3 \rangle$, $t = 1$

5. $\mathbf{r}(t) = \sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$, $t = \pi/4$

6. $\mathbf{r}(t) = e^t\,\mathbf{i} + e^{-t}\,\mathbf{j}$, $t = 0$

7. $\mathbf{r}(t) = e^{2t}\,\mathbf{i} + e^t\,\mathbf{j}$, $t = 0$

8. $\mathbf{r}(t) = (1 + \cos t)\,\mathbf{i} + (2 + \sin t)\,\mathbf{j}$, $t = \pi/6$

9–16 Find the derivative of the vector function.

9. $\mathbf{r}(t) = \langle t \sin t, t^2, t \cos 2t \rangle$

10. $\mathbf{r}(t) = \langle \tan t, \sec t, 1/t^2 \rangle$

11. $\mathbf{r}(t) = t\,\mathbf{i} + \mathbf{j} + 2\sqrt{t}\,\mathbf{k}$

12. $\mathbf{r}(t) = \dfrac{1}{1+t}\,\mathbf{i} + \dfrac{t}{1+t}\,\mathbf{j} + \dfrac{t^2}{1+t}\,\mathbf{k}$

13. $\mathbf{r}(t) = e^{t^2}\,\mathbf{i} - \mathbf{j} + \ln(1 + 3t)\,\mathbf{k}$

14. $\mathbf{r}(t) = at\cos 3t\,\mathbf{i} + b\sin^3 t\,\mathbf{j} + c\cos^3 t\,\mathbf{k}$

15. $\mathbf{r}(t) = \mathbf{a} + t\,\mathbf{b} + t^2\,\mathbf{c}$

16. $\mathbf{r}(t) = t\,\mathbf{a} \times (\mathbf{b} + t\,\mathbf{c})$

17–20 Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

17. $\mathbf{r}(t) = \langle te^{-t}, 2\arctan t, 2e^t \rangle$, $t = 0$

18. $\mathbf{r}(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4 \rangle$, $t = 1$

19. $\mathbf{r}(t) = \cos t\,\mathbf{i} + 3t\,\mathbf{j} + 2\sin 2t\,\mathbf{k}$, $t = 0$

20. $\mathbf{r}(t) = \sin^2 t\,\mathbf{i} + \cos^2 t\,\mathbf{j} + \tan^2 t\,\mathbf{k}$, $t = \pi/4$

21. If $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, find $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$, and $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

22. If $\mathbf{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t} \rangle$, find $\mathbf{T}(0)$, $\mathbf{r}''(0)$, and $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$.

23–26 Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

23. $x = 1 + 2\sqrt{t}$, $y = t^3 - t$, $z = t^3 + t$; $(3, 0, 2)$

24. $x = e^t$, $y = te^t$, $z = te^{t^2}$; $(1, 0, 0)$

25. $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = e^{-t}$; $(1, 0, 1)$

26. $x = \sqrt{t^2 + 3}$, $y = \ln(t^2 + 3)$, $z = t$; $(2, \ln 4, 1)$

27. Find a vector equation for the tangent line to the curve of intersection of the cylinders $x^2 + y^2 = 25$ and $y^2 + z^2 = 20$ at the point $(3, 4, 2)$.

28. Find the point on the curve $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, e^t \rangle$, $0 \leqslant t \leqslant \pi$, where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.

CAS **29–31** Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. Illustrate by graphing both the curve and the tangent line on a common screen.

29. $x = t$, $y = e^{-t}$, $z = 2t - t^2$; $(0, 1, 0)$

30. $x = 2\cos t$, $y = 2\sin t$, $z = 4\cos 2t$; $(\sqrt{3}, 1, 2)$

31. $x = t\cos t$, $y = t$, $z = t\sin t$; $(-\pi, \pi, 0)$

32. (a) Find the point of intersection of the tangent lines to the curve $\mathbf{r}(t) = \langle \sin \pi t, 2\sin \pi t, \cos \pi t \rangle$ at the points where $t = 0$ and $t = 0.5$.
    (b) Illustrate by graphing the curve and both tangent lines.

33. The curves $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t \rangle$ intersect at the origin. Find their angle of intersection correct to the nearest degree.