Applications of Quadric Surfaces

Examples of quadric surfaces can be found in the world around us. In fact, the world itself is a good example. Although the earth is commonly modeled as a sphere, a more accurate model is an ellipsoid because the earth's rotation has caused a flattening at the poles. (See Exercise 47.)

Circular paraboloids, obtained by rotating a parabola about its axis, are used to collect and reflect light, sound, and radio and television signals. In a radio telescope, for instance, signals from distant stars that strike the bowl are all reflected to the receiver at the focus and are therefore amplified. (The idea is explained in Problem 20 on page 271.) The same principle applies to microphones and satellite dishes in the shape of paraboloids.

Cooling towers for nuclear reactors are usually designed in the shape of hyperboloids of one sheet for reasons of structural stability. Pairs of hyperboloids are used to transmit rotational motion between skew axes. (The cogs of the gears are the generating lines of the hyperboloids. See Exercise 49.)

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A satellite dish reflects signals to the focus of a paraboloid.

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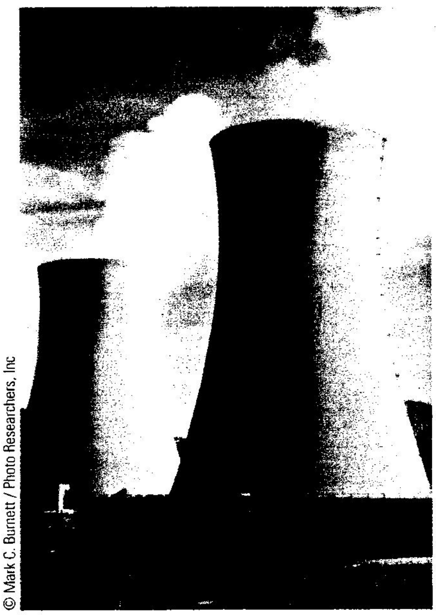
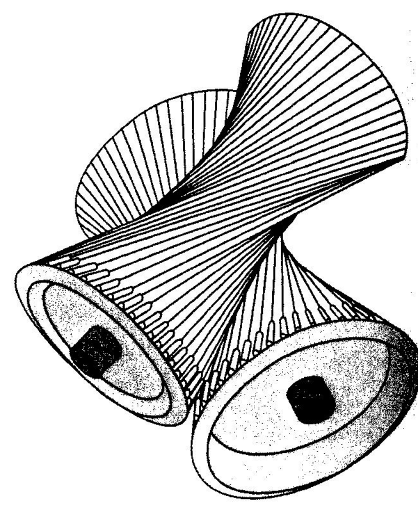


Nuclear reactors have cooling towers in the shape of hyperboloids.



Hyperboloids produce gear transmission.

12.6 Exercises

1. (a) What does the equation $y = x^2$ represent as a curve in $\mathbb{R}^2$?
   (b) What does it represent as a surface in $\mathbb{R}^3$?
   (c) What does the equation $z = y^2$ represent?

2. (a) Sketch the graph of $y = e^x$ as a curve in $\mathbb{R}^2$.
   (b) Sketch the graph of $y = e^x$ as a surface in $\mathbb{R}^3$.
   (c) Describe and sketch the surface $z = e^y$.

3–8 Describe and sketch the surface.

3. $x^2 + z^2 = 1$

4. $4x^2 + y^2 = 4$

5. $z = 1 - y^2$

6. $y = z^2$

7. $xy = 1$

8. $z = \sin y$

9. (a) Find and identify the traces of the quadric surface $x^2 + y^2 - z^2 = 1$ and explain why the graph looks like the graph of the hyperboloid of one sheet in Table 1.
   (b) If we change the equation in part (a) to $x^2 - y^2 + z^2 = 1$, how is the graph affected?
   (c) What if we change the equation in part (a) to $x^2 + y^2 + 2y - z^2 = 0$?

Graphing calculator or computer required   1. Homework Hints available at stewartcalculus.com

10. (a) Find and identify the traces of the quadric surface $-x^2 - y^2 + z^2 = 1$ and explain why the graph looks like the graph of the hyperboloid of two sheets in Table 1.
(b) If the equation in part (a) is changed to $x^2 - y^2 - z^2 = 1$, what happens to the graph? Sketch the new graph.

11–20 Use traces to sketch and identify the surface.

11. $x = y^2 + 4z^2$ **12.** $9x^2 - y^2 + z^2 = 0$

13. $x^2 = y^2 + 4z^2$ **14.** $25x^2 + 4y^2 + z^2 = 100$

15. $-x^2 + 4y^2 - z^2 = 4$ **16.** $4x^2 + 9y^2 + z = 0$

17. $36x^2 + y^2 + 36z^2 = 36$ **18.** $4x^2 - 16y^2 + z^2 = 16$

19. $y = z^2 - x^2$ **20.** $x = y^2 - z^2$

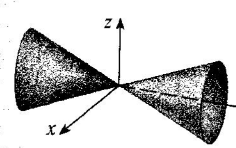
21–28 Match the equation with its graph (labeled I–VIII). Give reasons for your choice.

21. $x^2 + 4y^2 + 9z^2 = 1$ **22.** $9x^2 + 4y^2 + z^2 = 1$

23. $x^2 - y^2 + z^2 = 1$ **24.** $-x^2 + y^2 - z^2 = 1$

25. $y = 2x^2 + z^2$ **26.** $y^2 = x^2 + 2z^2$

27. $x^2 + 2z^2 = 1$ **28.** $y = x^2 - z^2$

I


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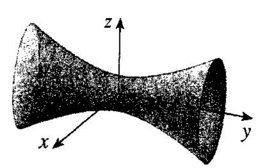

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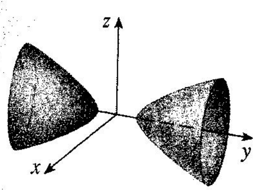

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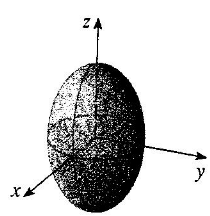

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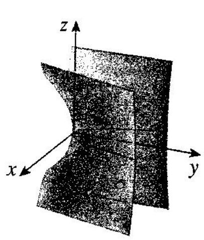

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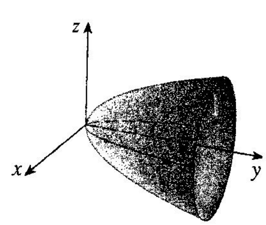

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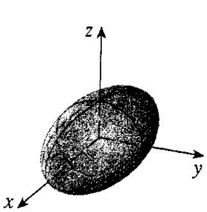

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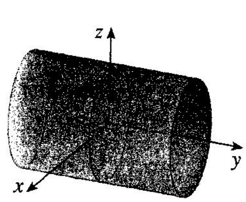

29–36 Reduce the equation to one of the standard forms, classify the surface, and sketch it.

29. $y^2 = x^2 + \frac{1}{9}z^2$ **30.** $4x^2 - y + 2z^2 = 0$

31. $x^2 + 2y - 2z^2 = 0$ **32.** $y^2 = x^2 + 4z^2 + 4$

33. $4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0$

34. $4y^2 + z^2 - x - 16y - 4z + 20 = 0$

35. $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$

36. $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$

37–40 Use a computer with three-dimensional graphing software to graph the surface. Experiment with viewpoints and with domains for the variables until you get a good view of the surface.

37. $-4x^2 - y^2 + z^2 = 1$ **38.** $x^2 - y^2 - z = 0$

39. $-4x^2 - y^2 + z^2 = 0$ **40.** $x^2 - 6x + 4y^2 - z = 0$

41. Sketch the region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 1$ for $1 \leqslant z \leqslant 2$.

42. Sketch the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.

43. Find an equation for the surface obtained by rotating the parabola $y = x^2$ about the y-axis.

44. Find an equation for the surface obtained by rotating the line $x = 3y$ about the x-axis.

45. Find an equation for the surface consisting of all points that are equidistant from the point $(-1, 0, 0)$ and the plane $x = 1$. Identify the surface.

46. Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane. Identify the surface.

47. Traditionally, the earth's surface has been modeled as a sphere, but the World Geodetic System of 1984 (WGS-84) uses an ellipsoid as a more accurate model. It places the center of the earth at the origin and the north pole on the positive z-axis. The distance from the center to the poles is 6356.523 km and the distance to a point on the equator is 6378.137 km.
(a) Find an equation of the earth's surface as used by WGS-84.
(b) Curves of equal latitude are traces in the planes $z = k$. What is the shape of these curves?
(c) Meridians (curves of equal longitude) are traces in planes of the form $y = mx$. What is the shape of these meridians?

48. A cooling tower for a nuclear reactor is to be constructed in the shape of a hyperboloid of one sheet (see the photo on page 832). The diameter at the base is 280 m and the minimum