

Comparing this equation with the standard form, we see that it is the equation of a sphere with center $(-2, 3, -1)$ and radius $\sqrt{8} = 2\sqrt{2}$.

EXAMPLE 7 What region in \mathbb{R}^3 is represented by the following inequalities?

$$1 \leq x^2 + y^2 + z^2 \leq 4 \quad z \leq 0$$

SOLUTION The inequalities

$$1 \leq x^2 + y^2 + z^2 \leq 4$$

can be rewritten as

$$1 \leq \sqrt{x^2 + y^2 + z^2} \leq 2$$

so they represent the points (x, y, z) whose distance from the origin is at least 1 and at most 2. But we are also given that $z \leq 0$, so the points lie on or below the xy -plane. Thus the given inequalities represent the region that lies between (or on) the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ and beneath (or on) the xy -plane. It is sketched in Figure 13.

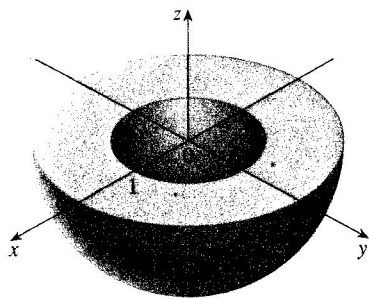


FIGURE 13

12.1 Exercises

- Suppose you start at the origin, move along the x -axis a distance of 4 units in the positive direction, and then move downward a distance of 3 units. What are the coordinates of your position?
- Sketch the points $(0, 5, 2)$, $(4, 0, -1)$, $(2, 4, 6)$, and $(1, -1, 2)$ on a single set of coordinate axes.
- Which of the points $A(-4, 0, -1)$, $B(3, 1, -5)$, and $C(2, 4, 6)$ is closest to the yz -plane? Which point lies in the xz -plane?
- What are the projections of the point $(2, 3, 5)$ on the xy -, yz -, and xz -planes? Draw a rectangular box with the origin and $(2, 3, 5)$ as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.
- Describe and sketch the surface in \mathbb{R}^3 represented by the equation $x + y = 2$.
- (a) What does the equation $x = 4$ represent in \mathbb{R}^2 ? What does it represent in \mathbb{R}^3 ? Illustrate with sketches.
(b) What does the equation $y = 3$ represent in \mathbb{R}^3 ? What does $z = 5$ represent? What does the pair of equations $y = 3$, $z = 5$ represent? In other words, describe the set of points (x, y, z) such that $y = 3$ and $z = 5$. Illustrate with a sketch.
- 7–8 Find the lengths of the sides of the triangle PQR . Is it a right triangle? Is it an isosceles triangle?
 - $P(3, -2, -3)$, $Q(7, 0, 1)$, $R(1, 2, 1)$
 - $P(2, -1, 0)$, $Q(4, 1, 1)$, $R(4, -5, 4)$
- Determine whether the points lie on a straight line.
 - $A(2, 4, 2)$, $B(3, 7, -2)$, $C(1, 3, 3)$
 - $D(0, -5, 5)$, $E(1, -2, 4)$, $F(3, 4, 2)$
- Find the distance from $(4, -2, 6)$ to each of the following.
 - The xy -plane
 - The yz -plane
 - The xz -plane
 - The x -axis
 - The y -axis
 - The z -axis
- Find an equation of the sphere with center $(-3, 2, 5)$ and radius 4. What is the intersection of this sphere with the yz -plane?
- Find an equation of the sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the coordinate planes.
- Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.
- Find an equation of the sphere that passes through the origin and whose center is $(1, 2, 3)$.
- 15–18 Show that the equation represents a sphere, and find its center and radius.
 - $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$
 - $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$
 - $2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$
 - $3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$