$$R_{100} \leq \frac{1}{2 \cdot 100^2} = 0.00005$$

Using a programmable calculator or a computer, we find that

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + 1} \approx \sum_{n=1}^{100} \frac{1}{n^3 + 1} \approx 0.6864538$$

with error less than 0.00005.

11.4 Exercises

1. Suppose $\Sigma\, a_n$ and $\Sigma\, b_n$ are series with positive terms and $\Sigma\, b_n$ is known to be convergent.
   (a) If $a_n > b_n$ for all n, what can you say about $\Sigma\, a_n$? Why?
   (b) If $a_n < b_n$ for all n, what can you say about $\Sigma\, a_n$? Why?

2. Suppose $\Sigma\, a_n$ and $\Sigma\, b_n$ are series with positive terms and $\Sigma\, b_n$ is known to be divergent.
   (a) If $a_n > b_n$ for all n, what can you say about $\Sigma\, a_n$? Why?
   (b) If $a_n < b_n$ for all n, what can you say about $\Sigma\, a_n$? Why?

3–32 Determine whether the series converges or diverges.

3. $\displaystyle \sum_{n=1}^{\infty} \frac{n}{2n^3 + 1}$

4. $\displaystyle \sum_{n=2}^{\infty} \frac{n^3}{n^4 - 1}$

5. $\displaystyle \sum_{n=1}^{\infty} \frac{n + 1}{n\sqrt{n}}$

6. $\displaystyle \sum_{n=1}^{\infty} \frac{n - 1}{n^2\sqrt{n}}$

7. $\displaystyle \sum_{n=1}^{\infty} \frac{9^n}{3 + 10^n}$

8. $\displaystyle \sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$

9. $\displaystyle \sum_{k=1}^{\infty} \frac{\ln k}{k}$

10. $\displaystyle \sum_{k=1}^{\infty} \frac{k \sin^2 k}{1 + k^3}$

11. $\displaystyle \sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3 + 4k + 3}}$

12. $\displaystyle \sum_{k=1}^{\infty} \frac{(2k - 1)(k^2 - 1)}{(k + 1)(k^2 + 4)^2}$

13. $\displaystyle \sum_{n=1}^{\infty} \frac{\arctan n}{n^{1.2}}$

14. $\displaystyle \sum_{n=2}^{\infty} \frac{\sqrt{n}}{n - 1}$

15. $\displaystyle \sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n - 2}$

16. $\displaystyle \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4 + 1}}$

17. $\displaystyle \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$

18. $\displaystyle \sum_{n=1}^{\infty} \frac{1}{2n + 3}$

19. $\displaystyle \sum_{n=1}^{\infty} \frac{1 + 4^n}{1 + 3^n}$

20. $\displaystyle \sum_{n=1}^{\infty} \frac{n + 4^n}{n + 6^n}$

21. $\displaystyle \sum_{n=1}^{\infty} \frac{\sqrt{n + 2}}{2n^2 + n + 1}$

22. $\displaystyle \sum_{n=3}^{\infty} \frac{n + 2}{(n + 1)^3}$

23. $\displaystyle \sum_{n=1}^{\infty} \frac{5 + 2n}{(1 + n^2)^2}$

24. $\displaystyle \sum_{n=1}^{\infty} \frac{n^2 - 5n}{n^3 + n + 1}$

25. $\displaystyle \sum_{n=1}^{\infty} \frac{\sqrt{n^4 + 1}}{n^3 + n^2}$

26. $\displaystyle \sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2 - 1}}$

27. $\displaystyle \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^2 e^{-n}$

28. $\displaystyle \sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

29. $\displaystyle \sum_{n=1}^{\infty} \frac{1}{n!}$

30. $\displaystyle \sum_{n=1}^{\infty} \frac{n!}{n^n}$

31. $\displaystyle \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

32. $\displaystyle \sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

33–36 Use the sum of the first 10 terms to approximate the sum of the series. Estimate the error.

33. $\displaystyle \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4 + 1}}$

34. $\displaystyle \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3}$

35. $\displaystyle \sum_{n=1}^{\infty} 5^{-n} \cos^2 n$

36. $\displaystyle \sum_{n=1}^{\infty} \frac{1}{3^n + 4^n}$

37. The meaning of the decimal representation of a number $0.d_1d_2d_3\ldots$ (where the digit d_i is one of the numbers 0, 1, 2, ..., 9) is that

$$0.d_1d_2d_3d_4\ldots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \frac{d_4}{10^4} + \cdots$$

Show that this series always converges.

Homework Hints available at stewartcalculus.com