11.2 Exercises

1. (a) What is the difference between a sequence and a series?
(b) What is a convergent series? What is a divergent series?

2. Explain what it means to say that $\sum_{n=1}^{\infty} a_n = 5$.

3–4 Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$ whose partial sums are given.

3. $s_n = 2 - 3(0.8)^n$

4. $s_n = \dfrac{n^2 - 1}{4n^2 + 1}$

5–8 Calculate the first eight terms of the sequence of partial sums correct to four decimal places. Does it appear that the series is convergent or divergent?

5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{1+\sqrt{n}}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$

9–14 Find at least 10 partial sums of the series. Graph both the sequence of terms and the sequence of partial sums on the same screen. Does it appear that the series is convergent or divergent? If it is convergent, find the sum. If it is divergent, explain why.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \cos n$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+4}}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{7^{n+1}}{10^n}$

13. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

14. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(n+2)}$

15. Let $a_n = \dfrac{2n}{3n+1}$.
(a) Determine whether $\{a_n\}$ is convergent.
(b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

16. (a) Explain the difference between

$$\sum_{i=1}^{n} a_i \quad \text{and} \quad \sum_{j=1}^{n} a_j$$

(b) Explain the difference between

$$\sum_{i=1}^{n} a_i \quad \text{and} \quad \sum_{i=1}^{n} a_j$$

17–26 Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

17. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$

18. $4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$

19. $10 - 2 + 0.4 - 0.08 + \cdots$

20. $2 + 0.5 + 0.125 + 0.03125 + \cdots$

21. $\displaystyle\sum_{n=1}^{\infty} 6(0.9)^{n-1}$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

24. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

25. $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

27–42 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

27. $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \cdots$

28. $\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$

30. $\displaystyle\sum_{k=1}^{\infty} \frac{k(k+2)}{(k+3)^2}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$

33. $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{2}$

34. $\displaystyle\sum_{n=1}^{\infty} [(0.8)^{n-1} - (0.3)^n]$

35. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1} \right)$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{2}{3}\right)^n}$

37. $\displaystyle\sum_{k=0}^{\infty} \left(\frac{\pi}{3} \right)^k$

38. $\displaystyle\sum_{k=1}^{\infty} (\cos 1)^k$

39. $\displaystyle\sum_{n=1}^{\infty} \arctan n$

40. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n} \right)$

41. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)} \right)$

42. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

43–48 Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum (as in Example 7). If it is convergent, find its sum.

43. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

44. $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$

45. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

46. $\displaystyle\sum_{n=1}^{\infty} \left(\cos \frac{1}{n^2} - \cos \frac{1}{(n+1)^2} \right)$

47. $\displaystyle\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)} \right)$

48. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^3-n}$