(c) (1) $\text{Dom } f = \mathbb{R} \setminus \{1\} = (-\infty, 1) \cup (1, \infty)$

(2) $$h'(x) = \frac{3(x+1)^2(x-1)^2 - 2(x+1)^3(x-1)}{(x-1)^4} =$$

$$= \frac{(x+1)^2(x-1)(3(x-1) - 2(x+1))}{(x-1)^4} = \frac{(x+1)^2(x-5)}{(x-1)^3}$$

$\nearrow \quad \nearrow \quad \searrow \quad \nearrow$

$+ \quad -1 \quad + \quad 1 \quad - \quad 5 \quad +$

$\Rightarrow h' \geq 0$ on $(-\infty, 1)$ and $(5, \infty)$, $h' < 0$ on $(1, 5)$ $\Rightarrow$ h is increasing on $(-\infty, 1)$ and $(5, \infty)$, h is decreasing on $(1, 5)$. By first derivative test h has local minimum at 5 ($h(5) = 13.5$), h has no local maxima (note $1 \notin \text{Dom } h$). Since h is defined on the union of open intervals, h has no global maxima (otherwise it would local max. as well). Also $\lim_{x \to -\infty} h(x) = -\infty$, so $\inf h = -\infty$, hense h has no global minima

(3) $$h''(x) = \frac{2(x+1)(x-5)(x-1)^3 + (x+1)^2(x-1)^3 - 3(x+1)^2(x-5)(x-1)^2}{(x-1)^6} =$$

$$= \frac{(x+1)(2(x-5)(x-1) + (x+1)(x-1) - 3(x+1)(x-5))}{(x-1)^4} = \frac{24(x+1)}{(x-1)^4}$$

$\cap \quad \cup \quad \cup$

$- \quad -1 \quad + \quad 1 \quad +$

Thus h is convex on $(-1,1)$ and $(1,\infty)$, h is concave on $(-\infty,-1)$, the inflection point is $(-1,0)$

(4) $\lim\limits_{x\to a} h(x) = h(a)$ for $\forall a \in \mathrm{Dom}\, h$,

$\lim\limits_{x\to 1^+} h(x) = \infty$, $\lim\limits_{x\to 1^-} h(x) = -\infty$ $\Rightarrow$

$x = 1$ is a vertical asymptote

$$\lim_{x\to\pm\infty} \frac{h(x)}{x} = \lim_{x\to\pm\infty} \frac{x^3+3x^2+3x+1}{x^3-2x^2+x} = 1$$

$$\lim_{x\to\pm\infty} (h(x)-x) = \lim_{x\to\pm\infty} \frac{5x^2+2x+1}{x^2-2x+1} = 5$$ Thus

$y = x+5$ is a slant asymptote (at ∞ and $-\infty$)

$\Rightarrow$ No horizontal asymptotes